Lösungen T 12

Aufgabe 1

a) $f(x) = -0{,}1x^4 + 0{,}1x^3 + 0{,}6x^2$

① $f'(x) = -0{,}4x^3 + 0{,}3x^2 + 1{,}2x$

$f''(x) = -1{,}2x^2 + 0{,}6x + 1{,}2$

$f'''(x) = -2{,}4x + 0{,}6$

② $x \to -\infty;\ f(x) = -\infty$

$x \to +\infty;\ f(x) = -\infty$

③ KS

④ $f(x) = 0$

$0 = -0{,}1x^4 + 0{,}1x^3 + 0{,}6x^2 \quad | :(-0{,}1)$

$0 = x^4 - x^3 - 6x^2$

$0 = x^2(x^2 - x - 6)$

$x^2 = 0 \quad | \sqrt{\ }$ $\qquad x^2 - x - 6 = 0$

$x_{1/2} = 0$ $\qquad x_{3/4} = 0{,}5 \pm \sqrt{0{,}25 + 6}$

$x_3 = 3$

$x_4 = -2$

$S_y(0|0)$

$S_{x_{1/2}}(0|0)$

$S_{x_3}(3|0)$

$S_{x_4}(-2|0)$

⑤ $f'(x) = 0$ und $f''(x) \neq 0$

$0 = -0{,}4x^3 + 0{,}3x^2 + 1{,}2x \quad | :(-0{,}4)$

$0 = x^3 - 0{,}75x^2 - 3x$

$0 = x(x^2 - 0{,}75x - 3)$

$x_1 = 0$ $\qquad x^2 - 0{,}75x - 3 = 0$

$x_{2/3} = +\frac{3}{8} \pm \sqrt{\frac{9}{64} + 3}$

$x_2 = 2{,}1$

$x_3 = -1{,}4$

$f''(0) = 1{,}2 > 0 \Rightarrow TP$

$f''(2{,}1) = -3{,}8 < 0 \Rightarrow HP$

$f''(-1{,}4) = -2{,}0 < 0 \Rightarrow HP$

$f(0) = 0 \quad TP(0|0)$

$f(2{,}1) = 1{,}6 \quad HP(2{,}1|1{,}6)$

$f(-1{,}4) = 0{,}5 \quad HP(-1{,}4|0{,}5)$

a) $f''(x) = 0$ und $f'''(x) \neq 0$

$0 = -1{,}2x^2 + 0{,}6x + 1{,}2 \quad | : (-1{,}2)$

$0 = x^2 - 0{,}5x - 1$

$x_{1/2} = +0{,}25 \pm \sqrt{0{,}0625 + 1}$

$x_1 = 1{,}3$

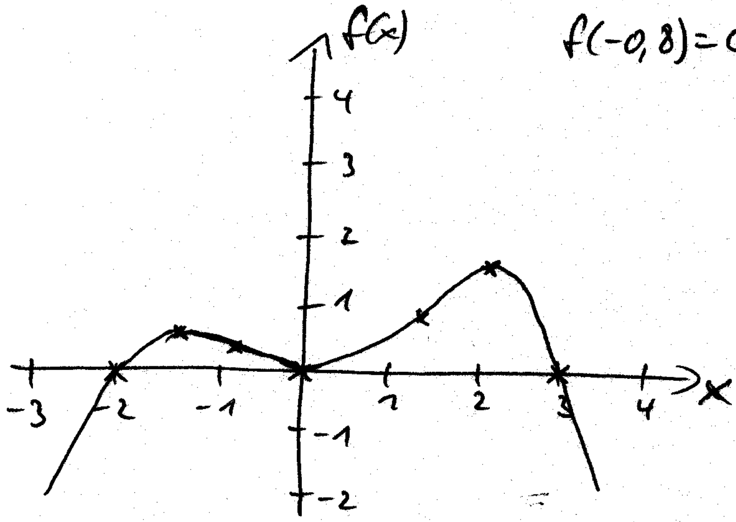
$x_2 = -0{,}8$

$f'''(1{,}3) = -2{,}5 < 0 \Rightarrow$ L-R-k

$f'''(-0{,}8) = 2{,}5 > 0 \Rightarrow$ R-L-k

$f(1{,}3) = 0{,}9 \quad W_{L-R}(1{,}3 \,|\, 0{,}9)$

$f(-0{,}8) = 0{,}3 \quad W_{R-L}(-0{,}8 \,|\, 0{,}3)$



b) $m = 0{,}4$

$f'(x) = m$

$0{,}4 = -0{,}4x^3 + 0{,}3x^2 + 1{,}2x \quad | -0{,}4$

$0 = -0{,}4x^3 + 0{,}3x^2 + 1{,}2x - 0{,}4 \quad | : (-0{,}4)$

$0 = x^3 - 0{,}75x^2 - 3x + 1$

$x_1 = 2$

$$
\begin{aligned}
&(x^3 - 0{,}75x^2 - 3x + 1) : (x - 2) = x^2 + 1{,}25x - 0{,}5 \\
&-(x^3 - 2x^2) \\
&\qquad 1{,}25x^2 - 3x \\
&\qquad -(1{,}25x^2 - 2{,}5x) \\
&\qquad\qquad -0{,}5x + 1 \\
&\qquad\qquad -(-0{,}5x + 1) \\
&\qquad\qquad\qquad 0
\end{aligned}
$$

$x^2 + 1{,}25x - 0{,}5 = 0$

$x_{2/3} = -\frac{5}{8} \pm \sqrt{\frac{25}{64} + 0{,}5}$

$$\boxed{\begin{aligned} x_2 &= 0{,}3 \\ x_3 &= -1{,}6 \end{aligned}}$$

$m = 0{,}4$
$x = 2 \Rightarrow f(2) = 1{,}6$ y

$t(x) = m \cdot x + b$
$1{,}6 = 0{,}4 \cdot 2 + b \quad | -0{,}8$
$0{,}8 = b$

$t(x) = 0{,}4x + 0{,}8$

c) $f(x) = t(x)$

$-0{,}1x^4 + 0{,}1x^3 + 0{,}6x^2 = 0{,}4x + 0{,}8 \quad | -0{,}4x - 0{,}8$

$-0{,}1x^4 + 0{,}1x^3 + 0{,}6x^2 - 0{,}4x - 0{,}8 = 0 \quad | : (-0{,}1)$

$0 = x^4 - x^3 - 6x^2 + 4x + 8$

$x_1 = 2$ (Tangente)

$(x^4 - x^3 - 6x^2 + 4x + 8) : (x-2) = x^3 + x^2 - 4x - 4$
$-(x^4 - 2x^3)$
$x^3 - 6x^2$
$-(x^3 - 2x^2)$
$-4x^2 + 4x$
$-(-4x^2 + 8x)$
$-4x + 8$
$-(-4x + 8)$
0

$x^3 + x^2 - 4x - 4 = 0$

$x_2 = 2$ (Tangente) Berühren!

$(x^3 + x^2 - 4x - 4) : (x-2) = x^2 + 3x + 2$
$-(x^3 - 2x^2)$
$3x^2 - 4x$
$-(3x^2 - 6x)$
$2x - 4$
$-(2x - 4)$
0

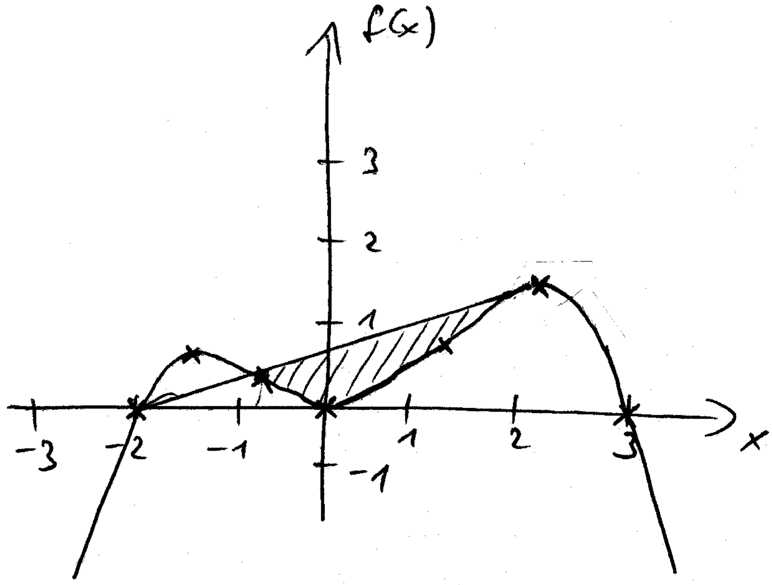
$x^2 + 3x + 2 = 0$

$x_{3/4} = -1{,}5 \pm \sqrt{2{,}25 - 2}$
$x_3 = -1$
$x_4 = -2$

$S_{1/2}(2|1{,}6)$ $S_3(-1|0{,}4)$
$S_4(-2|0)$

Da die weiteren Stellen mit der Steigung $m = 0{,}4$ bei $x = 0{,}3$ und $x = -1{,}6$ liegen und diese nicht mit den Schnittpunkten übereinstimmen, kann die Tangente am Linken Hügel nicht tangential anliegen.

d)



e) $a = -1$
$b = 2$

$$A = \left| \int_{-1}^{2} (-0{,}1x^4 + 0{,}1x^3 + 0{,}6x^2 - 0{,}4x - 0{,}8)\,dx \right|$$

$$= \left| \left[-\frac{1}{50}x^5 + \frac{1}{40}x^4 + 0{,}2x^3 - 0{,}2x^2 - 0{,}8x \right]_{-1}^{2} \right|$$

$$= \left| [-1{,}04] - [0{,}445] \right| = \left| [-1{,}485] \right|$$

$$= 1{,}485 \text{ FE}$$

Da 1 Einheit = 10 Meter und 1 FE = 100 m² sind, ist die Querschnittsfläche 148,5 m² groß.

f) Extremwertaufgabe: Differenz zwischen zwei Funktionen
t(x) liegt oberhalb f(x) ⇒ ① HB: $D = t(x) - f(x)$

② NB: $t(x) = 0{,}4x + 0{,}8$
$f(x) = -0{,}1x^4 + 0{,}1x^3 + 0{,}6x^2$

③ Da die Tangente bei −1 und 2 die Funktion f(x) schneidet, ist $\mathbb{D} = [-1; 2]$

④ $D(x) = 0{,}4x + 0{,}8 - (-0{,}1x^4 + 0{,}1x^3 + 0{,}6x^2)$
$D(x) = 0{,}4x + 0{,}8 + 0{,}1x^4 - 0{,}1x^3 - 0{,}6x^2$

$D(x) = 0{,}1x^4 - 0{,}1x^3 - 0{,}6x^2 + 0{,}4x + 0{,}8$ ZF

5 $D'(x) = 0{,}4x^3 - 0{,}3x^2 - 1{,}2x + 0{,}4$

$D''(x) = 1{,}2x^2 - 0{,}6x - 1{,}2$

$D'(x) = 0$ und $D''(x) \neq 0$

$0 = 0{,}4x^3 - 0{,}3x^2 - 1{,}2x + 0{,}4 \quad | : 0{,}4$

$0 = x^3 - 0{,}75x^2 - 3x + 1 \qquad x_1 = 2$

$(x^3 - 0{,}75x^2 - 3x + 1) : (x - 2)$ s. Aufgabe 1b)

$x_2 = 0{,}3 \qquad x_3 = -1{,}6$

$D''(2) = 2{,}4 > 0 \Rightarrow$ Min.

$D''(0{,}3) = -1{,}3 < 0 \Rightarrow$ Max $\Rightarrow x = 0{,}3$ LE

$D''(-1{,}6) = 2{,}8 > 0 \Rightarrow$ Min

6 $t(0{,}3) = 0{,}9$ LE

$f(0{,}3) = 0{,}1$ LE

7 $D = 0{,}9 - 0{,}1$

$D = 0{,}8$ LE

Da 1LE = 10m, beträgt die maximale Höhe 8m.

8 $D(-1) = 0 < 0{,}8$

$D(2) = 0 < 0{,}8$

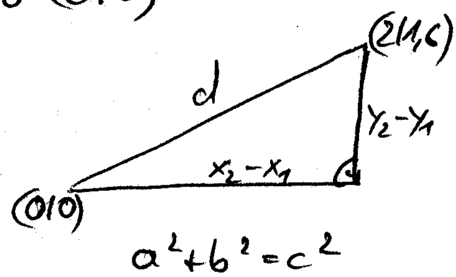
g) Anfang der Seilrutsche bei $(2|1{,}6)$

Talsenke = Tiefpunkt von $f(x)$ also $(0|0)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(2 - 0)^2 + (1{,}6 - 0)^2}$

$d = 2{,}56$ LE $\Rightarrow$ $\underline{\underline{25{,}6 \text{ m}}}$



$a^2 + b^2 = c^2$

Aufgabe 2

$$f(x) = \frac{2x-2}{x+1}$$

a) 1. $N(x) = 0$

$x + 1 = 0$

$x = -1 \Rightarrow \mathbb{D} = \mathbb{R} \setminus \{-1\}$

2. $f(x) = 0$ bzw. $Z(x) = 0$

$0 = 2x - 2$

$2 = 2x \quad |:2$

$x = 1$

$S_x(1|0)$

3. keine b.L.

4. $x = -1$ ist Pol

$$\left.\begin{aligned} &\text{l-Lim}_{x \to -1} \frac{2x-2}{x+1} = +\infty \\ &\text{r-Lim}_{x \to -1} \frac{2x-2}{x+1} = -\infty \end{aligned}\right\} \text{Pol mit VZW}$$

5. $Zg = Ng$

$$(2x-2):(x+1) = 2 - \frac{4}{x+1}$$

$$\underline{-(2x+2)}$$

$$-4$$

$\downarrow$

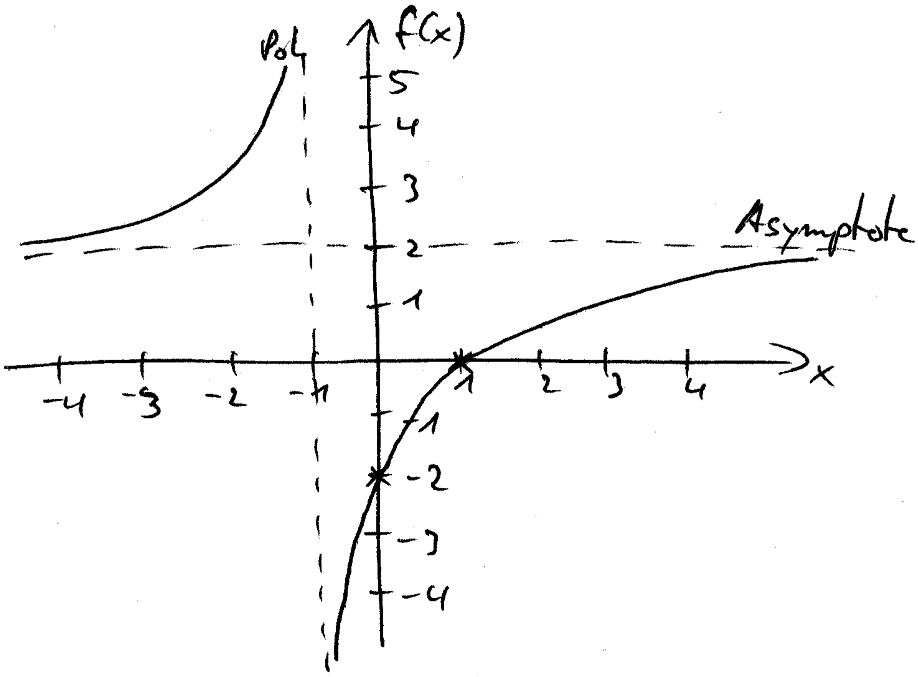
$y_A = 2$

6. $f(0) = -2$

$S_y(0|-2)$

7. KS

8. Skizze



b) $f(x) = g(x)$

$$\frac{2x-2}{x+1} = -2x+2 \quad | \cdot (x+1)$$

$$2x-2 = (-2x+2)\cdot(x+1)$$
$$2x-2 = -2x^2-2x+2x+2$$
$$2x-2 = -2x^2+2 \quad | -2x+2$$
$$0 = -2x^2-2x+4 \quad | :(-2)$$
$$0 = x^2+x-2$$
$$x_{1/2} = -0{,}5 \pm \sqrt{0{,}25+2}$$

$x_1 = 1$ $\quad g(1) = 0 \quad \underline{\underline{S_1(1|0)}}$

$x_2 = -2$ $\quad g(-2) = 6 \quad \underline{\underline{S_2(-2|6)}}$

Aufgabe 3

$f(x) = ax^3+bx^2+cx+d$

$f'(x) = 3ax^2+2bx+c$

$f''(x) = 6ax+2b$

$$\left.\begin{array}{ll} f(0)=2 & 2=d \\ f'(0)=0 & 0=c \end{array}\right\} \text{einsetzen}$$

$f''(1)=0 \quad 0=6a+2b$

$f'(1)=-3 \quad -3=3a+2b+c$

$$0 = 6a+2b$$
$$-3 = 3a+2b \quad | \cdot(-1)$$

$$\left.\begin{array}{l} 0 = 6a+2b \\ 3 = -3a-2b \end{array}\right] +$$

$$3 = 3a$$
$$1 = a$$

$$0 = 6\cdot 1+2b \quad |-6$$
$$-6 = 2b \quad |:2$$
$$-3 = b$$

$$\Rightarrow \underline{\underline{f(x) = x^3-3x^2+2}}$$