Lösungen S12

Aufgabe 1

a) $f(x) = -0{,}5x^4 + 1{,}5x^2 + 2$

① $f'(x) = -2x^3 + 3x$

$f''(x) = -6x^2 + 3$

$f'''(x) = -12x$

② $x \to -\infty; f(x) = -\infty$

$x \to +\infty; f(x) = -\infty$

③ AS

④ $f(x) = 0$

$0 = -0{,}5x^4 + 1{,}5x^2 + 2 \mid :(-0{,}5)$

$0 = x^4 - 3x^2 - 4$

$x^2 = z$

$0 = z^2 - 3z - 4$

$z_{1/2} = +1{,}5 \pm \sqrt{2{,}25+4}$

$z_1 = 4$

$z_2 = -1$

$z = x^2$

$x^2 = 4 \mid \sqrt{\ }$ $x_1 = 2$ $x_2 = -2$

$x^2 = -1 \mid \sqrt{\ }$ /

$S_{x_1}(2|0)$ $S_{x_2}(-2|0)$ $S_y(0|2)$

⑤ $f'(x) = 0$ und $f''(x) \neq 0$

$0 = -2x^3 + 3x$

$0 = x(-2x^2 + 3)$

$x_1 = 0$ $\quad -2x^2 + 3 = 0 \mid :(-2)$

$x^2 - 1{,}5 = 0$

$x^2 = 1{,}5 \mid \sqrt{\ }$

$x_2 = -1{,}2$

$x_3 = +1{,}2$

$f''(0) = 3 > 0 \Rightarrow TP$

$f''(-1{,}2) = -5{,}6 < 0 \Rightarrow HP$

$f''(1{,}2) = -5{,}6 < 0 \Rightarrow HP$

$f(0) = 2$

$f(-1{,}2) = 3{,}1$

$f(1{,}2) = 3{,}1$

HP (−1,2 | 3,1)

TP (0 | 2)

HP (1,2 | 3,1)

⑥ $f''(x)=0$ und $f'''(x)\neq 0$

$-6x^2+3=0$

$3=6x^2 \mid :6$

$0{,}5=x^2 \mid \sqrt{\ }$

$x_1=0{,}7$

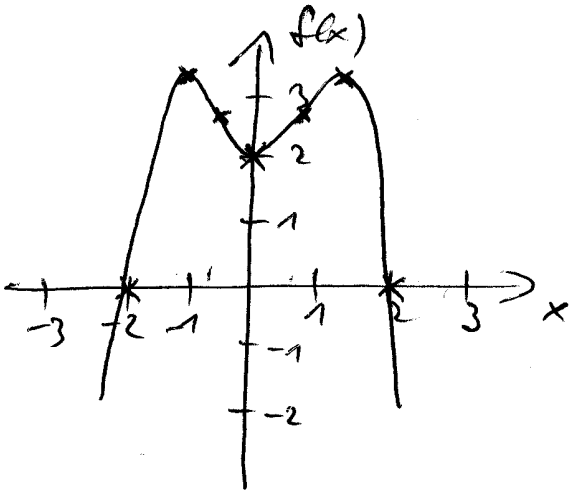
$x_2=-0{,}7$

$f'''(0{,}7)=-8{,}4<0 \Rightarrow L\text{-}R\text{-}K$

$f'''(-0{,}7)=8{,}4>0 \Rightarrow R\text{-}L\text{-}K$

$f(0{,}7)=2{,}6 \quad W_{L\text{-}R}(0{,}7\mid 2{,}6)$

$f(-0{,}7)=2{,}6 \quad W_{R\text{-}L}(-0{,}7\mid 2{,}6)$



b)

$$A=\int_{-2}^{2}(-0{,}5x^4+1{,}5x^2+2)\,dx=\left[-0{,}1x^5+0{,}5x^3+2x\right]_{-2}^{2}$$

$$=[4{,}8]-[-4{,}8]=\underline{\underline{9{,}6\ FE}}$$

c) $x=1$

$f(1)=3 \quad y$

$f'(1)=1 \quad m$

$t(x)=m\cdot x+b$

$3=1\cdot 1+b \mid -1$

$2=b$

$\underline{t(x)=x+2}$

$t(x)=f(x)$

$x+2=-0{,}5x^4+1{,}5x^2+2 \mid -x-2$

$0=-0{,}5x^4+1{,}5x^2-x \mid :(-0{,}5)$

$0=x^4-3x^2+2x$

$0=x(x^3-3x+2)$

$x_1=0$

$x^3+0x^2-3x+2=0$

$x_2 = 1$

$(x^3 + 0x^2 - 3x + 2) : (x-1) = x^2 + 1x - 2$

$-(x^3 - 1x^2)$

$+1x^2 - 3x$

$-(1x^2 - 1x)$

$-2x + 2$

$-(-2x + 2)$

0

$x^2 + 1x - 2 = 0$

$x_{3/4} = -0{,}5 \pm \sqrt{0{,}25 + 2}$

$x_3 = 1$

$x_4 = -2$

$f(0) = 2 \quad S_1(0|2)$

$f(1) = 3 \quad S_{2/3}(1|3)$

$f(-2) = 0 \quad S_4(-2|0)$

Aufgabe 2

$f(x) = \frac{-4}{-2-x}$

① $N(x) = 0$

$-2 - x = 0$

$-x = 2$

$x = -2 \Rightarrow \mathbb{D} = \mathbb{R} \setminus \{-2\}$

② $f(x) = 0$

$-4 \neq 0 \Rightarrow$ keine Nst.

③ keine behebbare Lücke

④ $x = -2$ ist Pol

$\lim\limits_{x \to -2,\ L} \frac{-4}{-2-x} = -\infty$

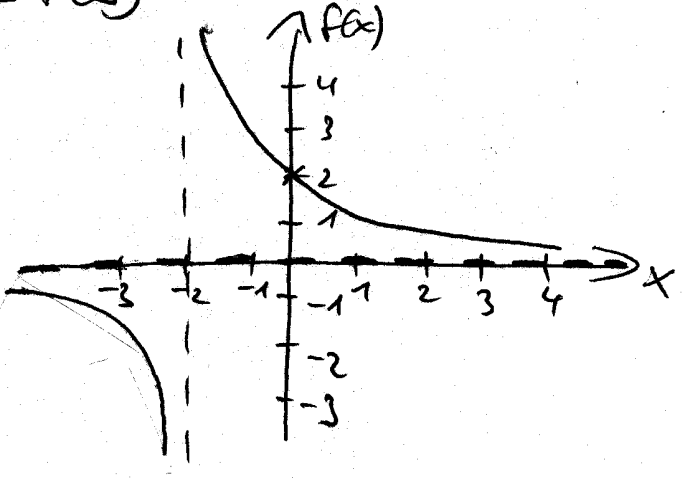
$\lim\limits_{x \to -2,\ r} \frac{-4}{-2-x} = +\infty$

} Pol mit VZW

⑤ $z_g < N_g \Rightarrow y_A = 0$

⑥ $f(0) = 2 \quad \underline{S_y(0|2)}$

⑦ KS



Aufgabe 3

$f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f(0) = -2 \quad -2 = d$

$f'(0) = 0 \quad 0 = c$ } einsetzen!

$f'(-1) = -1{,}5 \quad -1{,}5 = 3a - 2b + c$

$f(1) = 0 \quad 0 = a + b + c + d$

$-1{,}5 = 3a - 2b$

$0 = a + b - 2 \quad |+2$

$-1{,}5 = 3a - 2b$

$2 = a + b \quad |\cdot 2$

$-1{,}5 = 3a - 2b$

$4 = 2a + 2b$ ⊕

$2{,}5 = 5a$

$0{,}5 = a$

$2 = 0{,}5 + b \quad |-0{,}5$

$1{,}5 = b$

$$f(x) = 0{,}5x^3 + 1{,}5x^2 - 2$$

Aufgabe 4

a) $f(x) = -\frac{1}{8}x^4 + 1{,}5x^3 - 6x^2 + 9x$

$f'(x) = -\frac{1}{2}x^3 + 4{,}5x^2 - 12x + 9$

$f''(x) = -\frac{3}{2}x^2 + 9x - 12$

$f'''(x) = -3x + 9$

$f''(x) = 0$ und $f'''(x) \neq 0$

$0 = -\frac{3}{2}x^2 + 9x - 12 \quad | \cdot (-\frac{2}{3})$

$0 = x^2 - 6x + 8$

$x_{1/2} = +3 \pm \sqrt{9 - 8}$

$x_1 = 4$

$x_2 = 2$

$f'''(4) = -3 < 0 \Rightarrow L\text{-}R\text{-}K$

$f'''(2) = 3 > 0 \Rightarrow R\text{-}L\text{-}K$

$f(4) = 4 \qquad f(2) = 4$

$W_{L-R}(4|4) \qquad W_{R-L}(2|4)$

$t_1(x)$: $x = 4$

$y = 4$

$f'(4) = 1 = m$

$4 = 1 \cdot 4 + b \quad | -4$

$0 = b$

$\underline{t_1(x) = x}$

$t_2(x)$: $x = 2$

$y = 4$

$f'(2) = -1$

$4 = -1 \cdot 2 + b \quad | +2$

$6 = b$

$\underline{t_2(x) = -x + 6}$

b) $t_1(x) = t_2(x)$

$x = -x + 6 \quad | +x$

$2x = 6$

$x = 3$

$t_1(3) = 3$

$\underline{S(3|3)}$

c) $f'(x) = 0$ und $f''(x) \neq 0$

$0 = -\frac{1}{2}x^3 + 4{,}5x^2 - 12x + 9 \quad | \cdot (-\frac{2}{1})$

$0 = x^3 - 9x^2 + 24x - 18$ $\qquad x_1 = 3$

$(x^3 - 9x^2 + 24x - 18) : (x - 3) = x^2 - 6x + 6$

$-(x^3 - 3x^2)$

$-6x^2 + 24x$

$-(-6x^2 + 18x)$

$6x - 18$

$-(6x - 18)$

0

$x^2 - 6x + 6 = 0$

$x_{2/3} = +3 \pm \sqrt{9 - 6}$

$x_2 = 4{,}7$

$x_3 = 1{,}3$

$f''(3) = 1,5 > 0 \Rightarrow TP$

$f(3) = 3,4$

$TP(3|3,4)$

$f''(4,7) = -2,8 < 0 \Rightarrow HP$

$f''(1,3) = -2,8 < 0 \Rightarrow HP$

Nein, S stimmt nicht mit TP überein.

Aufgabe 5

$f(x) = x^3 - 3x^2 - 2x$

$f'(x) = 3x^2 - 6x - 2$

Stellen = x-Werte

$m = -2$

$f'(x) = m$

$-2 = 3x^2 - 6x - 2 \quad |+2$

$0 = 3x^2 - 6x \quad |:3$

$0 = x^2 - 2x$

$0 = x(x-2)$

$x_1 = 0$ $\qquad x - 2 = 0$

$x_2 = 2$

Aufgabe 6

a) $K(x) = x^3 - 15x^2 + 75x + 32$

$P(x) = -7x + 79$

$E(x) = -7x^2 + 79x$

$G(x) = E(x) - K(x)$

$= -7x^2 + 79x - (x^3 - 15x^2 + 75x + 32)$

$= -7x^2 + 79x - x^3 + 15x^2 - 75x - 32$

$\boxed{G(x) = -x^3 + 8x^2 + 4x - 32}$

$G(x) = 0$

$0 = -x^3 + 8x^2 + 4x - 32 \quad |:(-1)$

$0 = x^3 - 8x^2 - 4x + 32$

$x_1 = 2$ ME

GS

$(x^3 - 8x^2 - 4x + 32) : (x - 2) = x^2 - 6x - 16$

$-(x^3 - 2x^2)$

$-6x^2 - 4x$

$-(-6x^2 + 12x)$

$-16x + 32$

$-(-16x + 32)$

0

$x^2 - 6x - 16 = 0$

$x_{2/3} = 3 \pm \sqrt{9+16}$

$x_2 = 8$ ME GG

$[x_3 = -2]$

b) $G'(x) = -3x^2 + 16x + 4$

$G''(x) = -6x + 16$

$G'(x) = 0$ und $G''(x) \neq 0$

$0 = -3x^2 + 16x + 4 \quad | : (-3)$

$0 = x^2 - \frac{16}{3}x - \frac{4}{3}$

$x_{1/2} = \frac{8}{3} \pm \sqrt{\frac{64}{9} + \frac{4}{3}}$

$x_1 = 5{,}6$ ME x_{Gmax}

$[x_2 = -0{,}2]$

$P(5{,}6) = 39{,}8$ GE

$C(5{,}6 | 39{,}8)$

c) $K'(x) = 3x^2 - 30x + 75$

$K''(x) = 6x - 30$

$K'''(x) = 6$

$K''(x) = 0$ und $K'''(x) \neq 0$

$0 = 6x - 30$

$30 = 6x \quad | : 6$

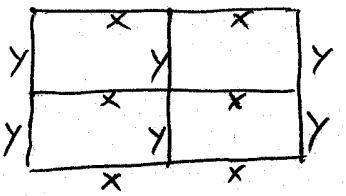
$x = 5$ ME

$K'''(5) = 6 > 0 \Rightarrow$ Minimum

$K'(5) = 0 \Rightarrow GK_{min}(5|10)$

An der Stelle $x = 5$ ME liegt die geringste Kostensteigerung mit 0GE vor.

Aufgabe 7



① HB: $A = 2x \cdot 2y$

② NB: $(584 + 4 \cdot 4 = 6x + 6y)$

$600 = 6x + 6y$

③ $6y = 600 - 6x \quad | : 6$

$y = 100 - x$

$y = 0$

$0 = 100 - x$

$x = 100 \quad \Rightarrow D = [0; 100]$

④

$A(x) = 2x \cdot 2(100 - x)$

$A(x) = 4x(100 - x)$

$A(x) = 400x - 4x^2$

$A(x) = -4x^2 + 400x$ zf.

⑤ $A'(x) = -8x + 400$

$A''(x) = -8$

$A'(x) = 0$ und $A''(x) \neq 0$

$0 = -8x + 400$ $A''(50) = -8 < 0$

$8x = 400$ $\Rightarrow$ Max.

$x = 50\,m$

⑥ $y = 100 - 50$

$y = 50\,m$

⑦ $A = 2 \cdot 50 \cdot 2 \cdot 50$

$A = 10.000\,m^2$

⑧ $A(0) = 0 < 10.000$

$A(100) = 0 < 10.000$