Lösungen PV 1 – 2012

Aufgabe 1

a) (1) $f(x) = \frac{1}{4}x^3 + x^2$

$f'(x) = \frac{3}{4}x^2 + 2x$

① $f''(x) = \frac{3}{2}x + 2$

$f'''(x) = \frac{3}{2}$

② $x \to -\infty;\ f(x) = -\infty$

$x \to +\infty;\ f(x) = +\infty$

③ KS

④ $S_y(0|0)$

$f(x) = 0$

$0 = \frac{1}{4}x^3 + x^2 \quad |:\frac{1}{4}$

$0 = x^3 + 4x^2$

$0 = x^2(x+4)$

$x_{1/2} = 0 \qquad x_3 = -4$

$S_{x_{1/2}}(0|0)$

$S_{x_3}(-4|0)$

⑤ $f'(x) = 0$ und $f''(x) \neq 0$

$0 = \frac{3}{4}x^2 + 2x \quad |:\frac{3}{4}$

$0 = x^2 + \frac{8}{3}x$

$0 = x(x + \frac{8}{3})$

$x_1 = 0 \qquad x_2 = -\frac{8}{3}$

$f''(0) = 2 > 0 \Rightarrow TP$

$f''(-\frac{8}{3}) = -2 < 0 \Rightarrow HP$

$f(0) = 0 \qquad \underline{TP(0|0)}$

$f(-\frac{8}{3}) = 2,4 \qquad \underline{HP(-2,7|2,4)}$

⑥ $f''(x) = 0$ und $f'''(x) \neq 0$

$0 = \frac{3}{2}x + 2 \quad |:\frac{3}{2}$

$0 = x + \frac{4}{3}$

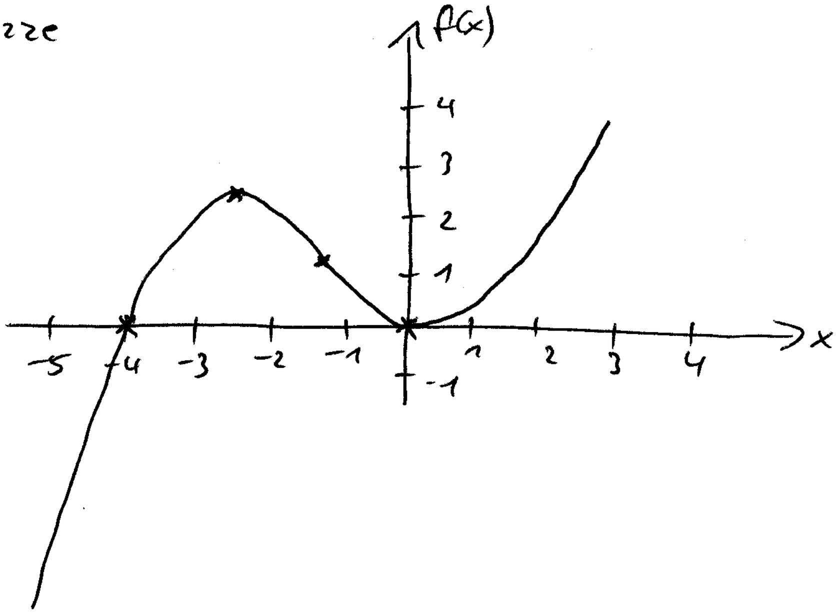
$x = -\frac{4}{3}$

$f'''(-\frac{4}{3}) = \frac{3}{2} > 0 \Rightarrow$ R-L-K

$f(-\frac{4}{3}) = 1,2$

$W_{R-L}\underline{(-1,3|1,2)}$

7 Skizze



(2) $f(x) = -x^3 + 2x^2 + 2{,}75x - 3{,}75$

$f'(x) = -3x^2 + 4x + 2{,}75$

① $f''(x) = -6x + 4$

$f'''(x) = -6$

② $x \to -\infty$; $f(x) = +\infty$ ③ KS

$x \to +\infty$; $f(x) = -\infty$

④ $S_y(0|-3{,}75)$ $f(x) = 0$ $0 = -x^3 + 2x^2 + 2{,}75x - 3{,}75 \;|:(-1)$

$0 = x^3 - 2x^2 - 2{,}75x + 3{,}75$

Auch bei Kommazahlen als Konstante sollte man die Teiler 1 und −1 ausprobieren.

$\Rightarrow x_1 = 1$ ist Teiler

$(x^3 - 2x^2 - 2{,}75x + 3{,}75) : (x-1) = x^2 - 1x - 3{,}75$

$-(x^3 - 1x^2)$

$-1x^2 - 2{,}75x$

$-(-1x^2 + 1x)$

$-3{,}75x + 3{,}75$

$-(-3{,}75x + 3{,}75)$

0

$x^2 - 1x - 3{,}75 = 0$

$x_{2/3} = +0{,}5 \pm \sqrt{0{,}25 + 3{,}75}$

$x_2 = 2{,}5$

$x_3 = -1{,}5$

$S_{x_1}(1|0)$ $S_{x_2}(2{,}5|0)$ $S_{x_3}(-1{,}5|0)$

③ PV1

⑤ $f'(x)=0$ und $f''(x)\neq 0$

$0=-3x^2+4x+2{,}75 \quad |:(-3)$

$0=x^2-\frac{4}{3}x-\frac{11}{12}$

$x_{1/2}=+\frac{2}{3}\pm\sqrt{\frac{4}{9}+\frac{11}{12}}$

$x_1=1{,}8 \qquad x_2=-0{,}5$

$f''(1{,}8)=-6{,}8<0 \Rightarrow HP$

$f''(-0{,}5)=7>0 \Rightarrow TP$

$f(1{,}8)=1{,}8 \qquad \underline{HP(1{,}8|1{,}8)}$

$f(-0{,}5)=-4{,}5 \qquad \underline{TP(-0{,}5|-4{,}5)}$

⑥ $f''(x)=0$ und $f'''(x)\neq 0$

$0=-6x+4 \quad |+6x$

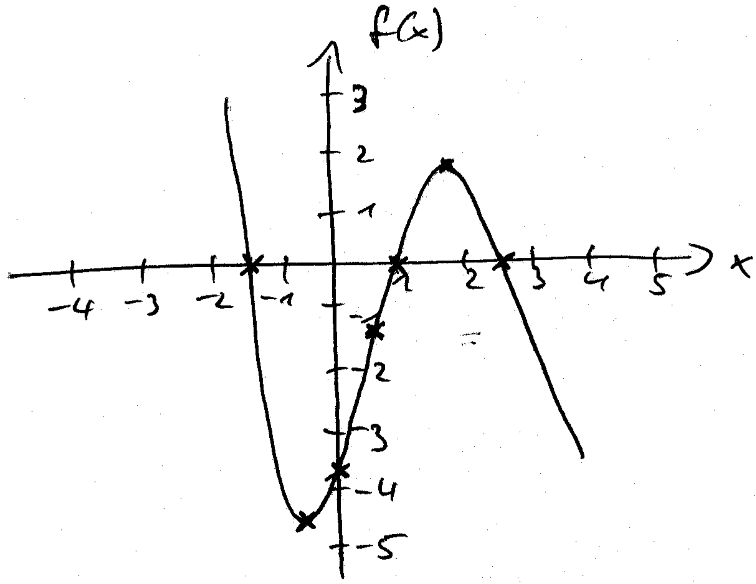
$6x=4 \quad |:6$

$x=\frac{2}{3}$

$f'''(\frac{2}{3})=-6<0 \Rightarrow L\text{-}R\text{-}K$

$f(\frac{2}{3})=-1{,}3 \qquad \underline{W_{L-R}(0{,}7|-1{,}3)}$

⑦ Skizze



(3)

$f(x)=-x^4+3x^2+4$

① $f'(x)=-4x^3+6x$

$f''(x)=-12x^2+6$

$f'''(x)=-24x$

② $x\to-\infty;\ f(x)=-\infty$

$x\to+\infty;\ f(x)=-\infty$

③ AS

④ $f(x)=0$ $\qquad \underline{S_y(0|4)}$

$0=-x^4+3x^2+4 \quad |:(-1)$

$0=x^4-3x^2-4$

$x^2=z$

$0=z^2-3z-4$

$z_{1/2}=+1{,}5\pm\sqrt{2{,}25+4}$

$z_1=4 \quad | \quad z=x^2 \quad x^2=4 \ |\sqrt{\ }$

$z_2=-1 \qquad\qquad \cancel{x^2=-1 \ |\sqrt{\ }}$

$x_1=2 \qquad x_2=-2$

$\underline{S_{x_1}(2|0)} \qquad \underline{S_{x_2}(-2|0)}$

⑤ $f'(x) = 0$ und $f''(x) \neq 0$

④ PV_1

$0 = -4x^3 + 6x \mid :(-4)$

$0 = x^3 - 1{,}5x$

$0 = x(x^2 - 1{,}5)$

$\underline{x_1 = 0}$ $\quad x^2 - 1{,}5 = 0$

$x^2 = 1{,}5 \mid \sqrt{\ }$

$\underline{x_2 = 1{,}2}$

$\underline{x_3 = -1{,}2}$

$f''(0) = 6 > 0 \Rightarrow TP$

$f''(1{,}2) = -11{,}3 < 0 \Rightarrow HP$

$f''(-1{,}2) = -11{,}3 < 0 \Rightarrow HP$

$f(0) = 4$ $\quad \underline{TP(0|4)}$

$f(1{,}2) = 6{,}2$ $\quad \underline{HP(1{,}2|6{,}2)}$

$f(-1{,}2) = 6{,}2$ $\quad \underline{HP(-1{,}2|6{,}2)}$

⑥ $f''(x) = 0$ und $f'''(x) \neq 0$

$0 = -12x^2 + 6 \mid +12x^2$

$12x^2 = 6 \quad \mid :12$

$x^2 = 0{,}5$

$x_1 = 0{,}7$

$x_2 = -0{,}7$

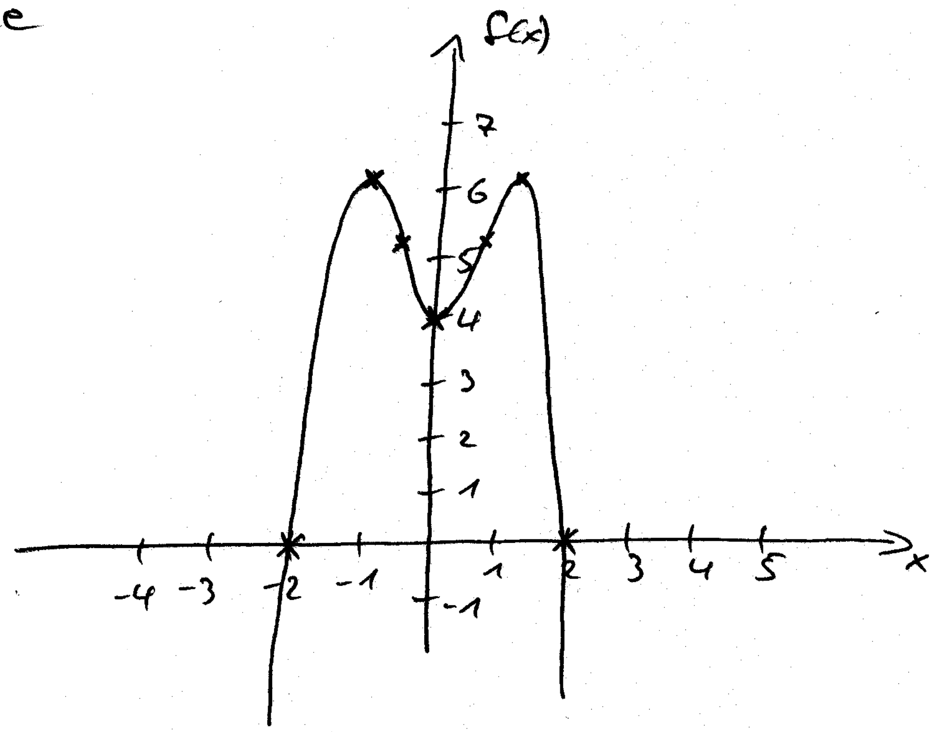
$f'''(0{,}7) = -16{,}8 < 0 \Rightarrow L\text{-}R\text{-}k$

$f'''(-0{,}7) = +16{,}8 > 0 \Rightarrow R\text{-}L\text{-}k$

$f(0{,}7) = 5{,}2$ $\quad W_{L-R}(0{,}7|5{,}2)$

$f(-0{,}7) = 5{,}2$ $\quad W_{R-L}(-0{,}7|5{,}2)$

⑦ Skizze



b) (1)

$$A = \int_{-4}^{0} \left(\frac{1}{4}x^3 + x^2\right)dx = \left[\frac{1}{16}x^4 + \frac{1}{3}x^3\right]_{-4}^{0}$$

$$= [0] - [-5\tfrac{1}{3}] = \underline{\underline{5\tfrac{1}{3}\ FE}}$$

(2)

$$A_1 = \left|\int_{-1,5}^{1} (-x^3 + 2x^2 + 2{,}75x - 3{,}75)\,dx\right|$$

$$= \left|\left[-\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{11}{8}x^2 - 3{,}75x\right]_{-1,5}^{1}\right|$$

$$= \left|[-2{,}0] - [5{,}2]\right| = |-7{,}2| = 7{,}2\ FE$$

$$A_2 = \int_{1}^{2,5} (-x^3 + 2x^2 + 2{,}75x - 3{,}75)\,dx$$

$$= \left[-\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{11}{8}x^2 - 3{,}75x\right]_{1}^{2,5}$$

$$= [-0{,}1] - [-2{,}0] = 1{,}9\ FE$$

$$A_{gesamt} = A_1 + A_2$$

$$= 7{,}2 + 1{,}9 = \underline{\underline{9{,}1\ FE}}$$

(3)

As!

$$A = 2 \cdot \int_{0}^{2} (-x^4 + 3x^2 + 4)\,dx$$

$$= 2 \cdot \left[-\frac{1}{5}x^5 + x^3 + 4x\right]_{0}^{2}$$

$$= 2 \cdot ([9{,}6] - [0]) = 2 \cdot 9{,}6 = \underline{\underline{19{,}2\ FE}}$$

c) $x = 2$ $\quad f(2) = 6\ (y)$ $\quad f'(2) = 7\ (m)$

$t(x) = m \cdot x + b$ $\quad 6 = 7 \cdot 2 + b \quad |-14$

$-8 = b \quad \Rightarrow \underline{\underline{t(x) = 7x - 8}}$

d) $m=4 \quad f'(x)=m$

$4=-3x^2+4x+2{,}75 \quad |-4$

$0=-3x^2+4x-1{,}25 \quad |:(-3)$

$0=x^2-\frac{4}{3}x+\frac{5}{12}$

$x_{1/2}=+\frac{2}{3}\pm\sqrt{\frac{4}{9}-\frac{5}{12}}$

$\underline{\underline{x_1=0{,}8}}$

$\underline{\underline{x_2=0{,}5}}$

e) HP$(1{,}2|6{,}2)$ $\quad t(x)$ im HP hat $m=0$!

$y=6{,}2 \quad x=1{,}2 \Rightarrow 6{,}2=0\cdot 1{,}2+b$

$6{,}2=b$

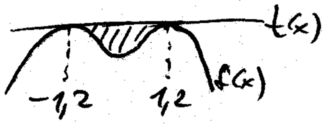
$\Rightarrow \underline{\underline{t(x)=6{,}2}}$

$t(x)=f(x)$

$6{,}2=-x^4+3x^2+4 \quad |-6{,}2$

$\boxed{0=-x^4+3x^2-2{,}2}$ Das ist die Funktion zum Aufleiten. Die Grenzen sind die beiden Hochpunkte $\Rightarrow$

$a=-1{,}2 \quad b=1{,}2$



$A=\int_{-1{,}2}^{1{,}2}$ oder $A=2\cdot\int_0^{1{,}2}$ weil $\underline{AS}$

$A=\left|\int_{-1{,}2}^{1{,}2}(-x^4+3x^2-2{,}2)\,dx\right|=\left|\left[-\frac{1}{5}x^5+x^3-2{,}2x\right]_{-1{,}2}^{1{,}2}\right|$

$=|[-1{,}4]-[1{,}4]|=|-2{,}8|=\underline{\underline{2{,}8\text{ FE}}}$

Aufgabe 2

a) $f_1(x) = f_2(x)$

(1) $x^3 + 1{,}5x^2 + 4 = 9x \quad |-9x$

$x^3 + 1{,}5x^2 - 9x + 4 = 0 \qquad \underline{x_1 = 2}$

$(x^3 + 1{,}5x^2 - 9x + 4) : (x-2) = x^2 + 3{,}5x - 2$
$-(x^3 - 2x^2)$
$3{,}5x^2 - 9x$
$-(3{,}5x^2 - 7x)$
$-2x + 4$
$-(-2x + 4)$
0

$x^2 + 3{,}5x - 2 = 0$

$x_{2/3} = -1{,}75 \pm \sqrt{3{,}0625 + 2}$

$\underline{x_2 = 0{,}5}$

$\underline{x_3 = -4}$

$f_2(2) = 18 \qquad \underline{\underline{S_1(2|18)}}$

$f_2(0{,}5) = 4{,}5 \qquad \underline{\underline{S_2(0{,}5|4{,}5)}}$

$f_2(-4) = -36 \qquad \underline{\underline{S_3(-4|-36)}}$

(2) $f_1(x) = f_2(x)$

$0{,}2x^3 + 0{,}6x^2 - 2{,}6x - 3 = 2x^2 + 12x + 10 \quad |-2x^2 - 12x - 10$

$0{,}2x^3 - 1{,}4x^2 - 14{,}6x - 13 = 0 \quad |:0{,}2$

$x^3 - 7x^2 - 73x - 65 = 0 \qquad \underline{x_1 = -1}$

$(x^3 - 7x^2 - 73x - 65) : (x+1) = x^2 - 8x - 65$
$-(x^3 + 1x^2)$
$-8x^2 - 73x$
$-(-8x^2 - 8x)$
$-65x - 65$
$-(-65x - 65)$
0

$x^2 - 8x - 65 = 0$

$x_{2/3} = +4 \pm \sqrt{16 + 65}$

$\underline{x_2 = 13}$

$\underline{x_3 = -5}$

$f_2(-1) = 0$ $\underline{\underline{S_1(-1|0)}}$

$f_2(13) = 504$ $\underline{\underline{S_2(13|504)}}$

$f_2(-5) = 0$ $\underline{\underline{S_3(-5|0)}}$

b)

(1)

$$A_1 = \int_{-4}^{0,5} (x^3 + 1{,}5x^2 - 9x + 4)\,dx$$

$$= \left[\tfrac{1}{4}x^4 + 0{,}5x^3 - 4{,}5x^2 + 4x\right]_{-4}^{0,5}$$

$$= [1{,}0] - [-56] = \underline{\underline{57\,FE}}$$

$$A_2 = \left|\int_{0,5}^{2} (x^3 + 1{,}5x^2 - 9x + 4)\,dx\right|$$

$$= \left|\left[\tfrac{1}{4}x^4 + 0{,}5x^3 - 4{,}5x^2 + 4x\right]_{0,5}^{2}\right|$$

$$= |[-2] - [1{,}0]| = |-3| = \underline{\underline{3\,FE}}$$

$$A_{gesamt} = A_1 + A_2$$

$$= 57 + 3 = \underline{\underline{60\,FE}}$$

(2)

$$A_1 = \int_{-5}^{-1} (0{,}2x^3 - 1{,}4x^2 - 14{,}6x - 13)\,dx$$

$$= \left[\tfrac{1}{20}x^4 - \tfrac{7}{15}x^3 - 7{,}3x^2 - 13x\right]_{-5}^{-1}$$

$$= [6{,}2] - [-22{,}1] = \underline{\underline{28{,}3\,FE}}$$

$$A_2 = \left|\int_{-1}^{13} (0{,}2x^3 - 1{,}4x^2 - 14{,}6x - 13)\,dx\right|$$

$$= \left|\left[\tfrac{1}{20}x^4 - \tfrac{7}{15}x^3 - 7{,}3x^2 - 13x\right]_{-1}^{13}\right|$$

$$= |[-999{,}9] - [6{,}2]| = |-1006{,}1| = 1006{,}1\,FE$$

$$A_{gesamt} = A_1 + A_2$$
$$= 28{,}3 + 1006{,}1 = \underline{\underline{1034{,}4 \text{ FE}}}$$

Aufgabe 3

a) $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$f(5) = 0$	$0 = 125a + 25b + 5c + d$
$f'(5) = 0$	$0 = 75a + 10b + c$
$f(3) = -1$	$-1 = 27a + 9b + 3c + d$
$f''(3) = 0$	$0 = 18a + 2b$

b) $f(x) = ax^4 + bx^2 + c$

$f'(x) = 4ax^3 + 2bx$

$f(0) = 3$	$3 = c$
$f(-1) = 5$	$5 = a + b + c$
$f'(-1) = 0$	$0 = -4a - 2b$

c einsetzen

umstellen

c) $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f'(-1) = 0$	$0 = 3a - 2b + c$
$f(-2) = 0$	$0 = -8a + 4b - 2c + d$
$f(4) = 7$	$7 = 64a + 16b + 4c + d$
$f'(4) = 10$	$10 = 48a + 8b + c$

Aufgabe 4

$5 = a + b + 3 \quad | -3$

$2 = a + b \quad | \cdot 2$

$4 = 2a + 2b$

$0 = -4a - 2b$ (+)

$4 = -2a$

$-2 = a$

a einsetzen

$2 = -2 + b \quad | + 2$

$4 = b$

$\Rightarrow f(x) = \underline{\underline{-2x^4 + 4x^2 + 3}}$