Lösungen zu PV 1

1.) a) (1) $f(x) = \frac{1}{4}x^3 + x^2$

$f'(x) = \frac{3}{4}x^2 + 2x$

$f''(x) = \frac{3}{2}x + 2$

$f'''(x) = \frac{3}{2}$

1. Verlauf ~
2. Sym KS
3. S_x/S_y $S_y(0|0)$

$f(x) = 0$

$0 = \frac{1}{4}x^3 + x^2$

$0 = x^2\left(\frac{1}{4}x + 1\right)$

$x_{1/2} = 0$ $S_{x_{1/2}}(0|0)$

$\frac{1}{4}x + 1 = 0$

$x = -4$ $S_{x_3}(-4|0)$

4. Ext.

$f'(x) = 0$ $f''(x) \neq 0$

$0 = \frac{3}{4}x^2 + 2x$

$0 = x\left(\frac{3}{4}x + 2\right)$

$x_1 = 0$ $\frac{3}{4}x + 2 = 0$

$x_2 = -\frac{8}{3}$

$f''(0) = +2 > 0 \Rightarrow TP$ $f''\left(-\frac{8}{3}\right) = -2 < 0 \Rightarrow HP$

$f(0) = 0$ $f\left(-\frac{8}{3}\right) = 2{,}4$

$-2{,}7$

$TP(0|0)$ $H(-2{,}7|2{,}4)$

5. Wendep.

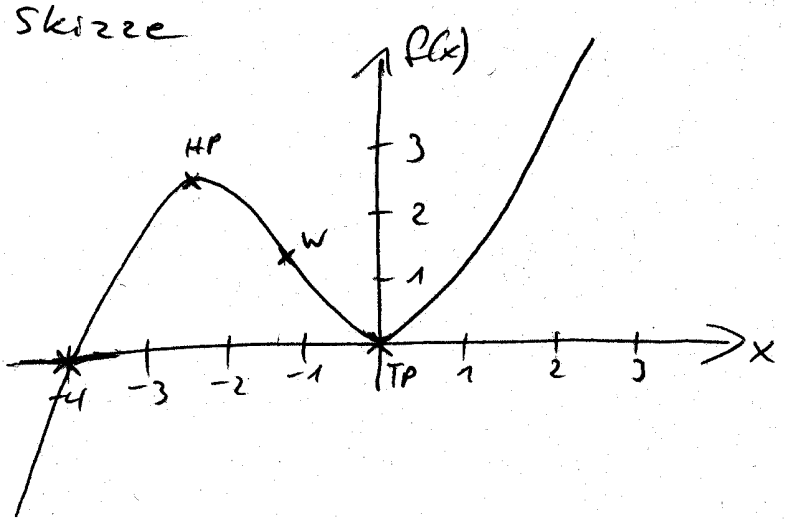
$f''(x) = 0 \quad f'''(x) \neq 0$

$0 = \frac{3}{2}x + 2$

$x = -\frac{4}{3}$ $\qquad f'''(-\frac{4}{3}) = \frac{3}{2} > 0 \Rightarrow$ R-L

$f(-\frac{4}{3}) = 1{,}2$ $\qquad W_{R-L}(-1{,}3 \mid 1{,}2)$

6. Skizze



(2) $f(x) = -x^3 + 2x^2 + 2{,}75x - 3{,}75$

$f'(x) = -3x^2 + 4x + 2{,}75$

$f''(x) = -6x + 4$

$f'''(x) = -6$

1. Verlauf ∩

2. Sym KS

3. S_x / S_y $\qquad S_y(0 \mid -3{,}75)$

$f(x) = 0$

$0 = -x^3 + 2x^2 + 2{,}75x - 3{,}75 \quad | \cdot (-1)$

$0 = x^3 - 2x^2 - 2{,}75x + 3{,}75$ $\qquad x_1 = 1$

Polynomdivision

$(x^3 - 2x^2 - 2{,}75x + 3{,}75) : (x-1) = x^2 - 1x - 3{,}75$

$-(x^3 - 1x^2)$

$-1x^2 - 2{,}75x$

$-(-1x^2 + 1x)$

$-3{,}75x + 3{,}75$

$-(-3{,}75x + 3{,}75)$

0

$x^2 - 1x - 3{,}75 = 0$

$x_{2/3} = +0{,}5 \pm \sqrt{0{,}25 + 3{,}75}$

$= 0{,}5 \pm 2$

$x_2 = 2{,}5$

$x_3 = -1{,}5$

$S_{x_1}(1|0) \quad S_{x_2}(2{,}5|0) \quad S_{x_3}(-1{,}5|0)$

4. Ext.

$f'(x) = 0 \quad f''(x) \neq 0$

$0 = -3x^2 + 4x + 2{,}75 \mid :(-3)$

$0 = x^2 - \frac{4}{3}x - \frac{11}{12}$

$x_{1/2} = +\frac{2}{3} \pm \sqrt{\frac{4}{9} + \frac{11}{12}}$

$= +\frac{2}{3} \pm \frac{7}{6}$

$x_1 = \frac{11}{6} \quad x_2 = -0{,}5$

$[= 1{,}8]$

$0 = x^2 - 1{,}3x - 0{,}9$

$x_{1/2} = +0{,}65 \pm \sqrt{0{,}4225 + 0{,}9}$

$= +0{,}65 \pm 1{,}15$

$x_1 = 1{,}8 \quad x_2 = -0{,}5$

$f''(1{,}8) = -5 < 0 \Rightarrow HP \qquad f(1{,}8) = 1{,}8 \quad H(1{,}8|1{,}8)$

$f''(-0{,}5) = 7 > 0 \Rightarrow TP \qquad f(-0{,}5) = -6 \quad T(-0{,}5|-6)$

5. Wendep.

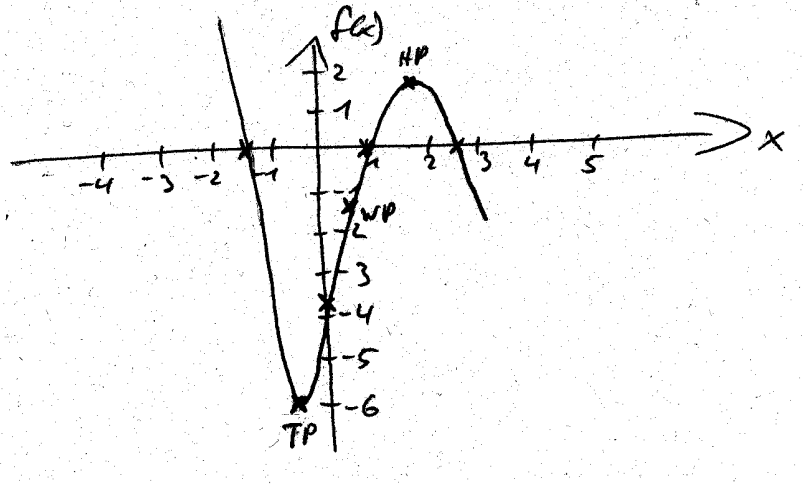
$f''(x) = 0 \qquad f'''(x) \neq 0$

$0 = -6x + 4 \qquad f'''(\frac{2}{3}) = -6 < \Rightarrow L\text{-}R$

$x = \frac{2}{3}$

$f(\frac{2}{3}) = -1{,}3 \qquad W_{L\text{-}R}(0{,}7|-1{,}3)$

6. Skizze



(3) $f(x) = -x^4 + 3x^2 + 4$

$f'(x) = -4x^3 + 6x$

$f''(x) = -12x^2 + 6$

$f'''(x) = -24x$

1. Verlauf ∩ 2. Sym AS

3. S_x / S_y $S_y(0|4)$

$f(x) = 0$ $\quad 0 = -x^4 + 3x^2 + 4 \quad | \cdot (-1)$

$0 = x^4 - 3x^2 - 4$

$x^2 = z$

$0 = z^2 - 3z - 4$

$z_{1/2} = +1{,}5 \pm \sqrt{2{,}25 + 4}$

$z_1 = 4 \qquad z_2 = -1$

$z = x^2$

$x^2 = 4$ ✓ $\qquad x^2 = -1$ / ✓

$x_1 = 2$

$x_2 = -2$

$S_{x_1}(2|0)$

$S_{x_2}(-2|0)$

4. Extr.

$f'(x) = 0 \qquad f''(x) \neq 0$

$0 = -4x^3 + 6x$

$0 = x(-4x^2 + 6)$

$x_1 = 0$

$-4x^2 + 6 = 0$

$x^2 = 1{,}5$

$x_2 = 1{,}2$

$x_3 = -1{,}2$

$f''(0) = 6 > 0 \Rightarrow TP$

$f''(1{,}2) = -11{,}3 < 0 \Rightarrow HP$

$f''(-1{,}2) = -11{,}3 < 0 \Rightarrow HP$

$f(0) = 4$

$f(1{,}2) = 6{,}2$

$f(-1{,}2) = 6{,}2$

$H(-1{,}2|6{,}2)$

$T(0|4)$

$H(1{,}2|6{,}2)$

5. Wendep.

$f'''(x) = 0 \qquad f'''(x) \neq 0$

$0 = -12x^2 + 6$

$x^2 = 0{,}5 \qquad x_1 = +0{,}7 \qquad x_2 = +0{,}7$

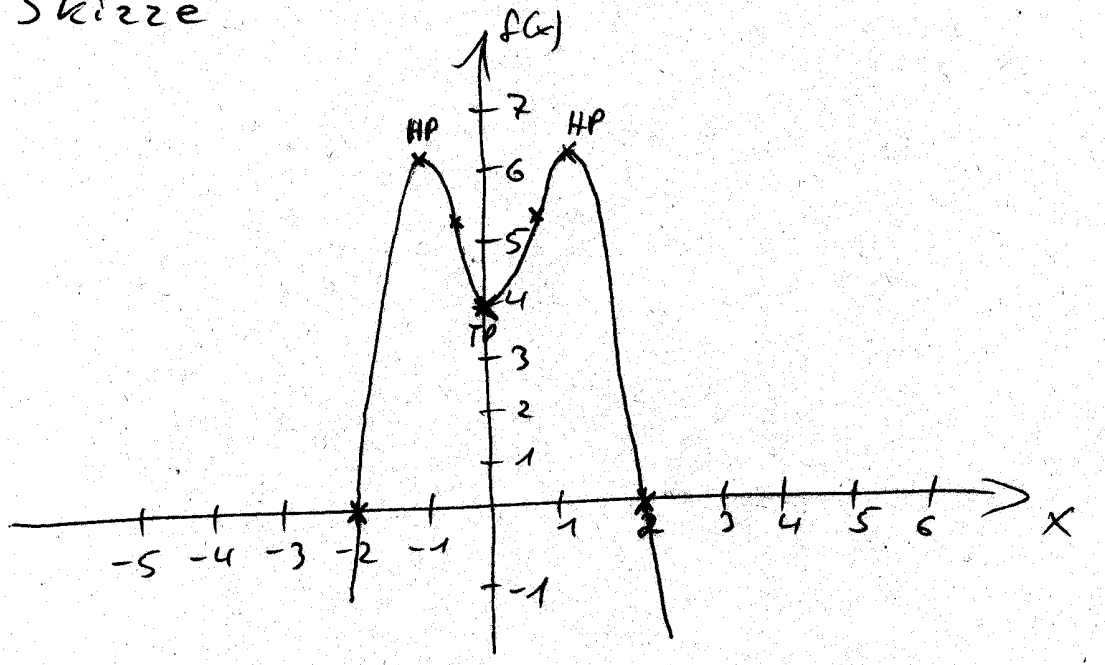
$f''(0,7) = -16,8 < 0 \Rightarrow L-R$

$f''(-0,7) = +16,8 > 0 \Rightarrow R-L$

$f(0,7) = 5,2 \qquad W_{L-R}(0,7|5,2)$

$f(-0,7) = 5,2 \qquad W_{R-L}(-0,7|5,2)$

6. Skizze



1.) b)

(1) $x_1 = 0 \qquad x_2 = -4$

$$A = \int_{-4}^{0} \left(\frac{1}{4}x^3 + x^2\right) dx = \left[\frac{1}{16}x^4 + \frac{1}{3}x^3\right]_{-4}^{0}$$

$$= [0] - \left[-5\tfrac{1}{3}\right] = \underline{5\tfrac{1}{3}\ FE}$$

(2) $x_1 = 1 \qquad x_2 = 2,5 \qquad x_3 = -1,5$

$$A_1 = \left|\int_{-1,5}^{1} (-x^3 + 2x^2 + 2,75x - 3,75)\,dx\right| = \left|\left[-\frac{1}{4}x^4 + \frac{2}{3}x^3 + 1,375x^2 - 3,75x\right]_{-1,5}^{1}\right|$$

$$= \left|\left[-\frac{7}{12}\right] - \left[5\,\frac{13}{64}\right]\right| = |-5,8| = 5,8\ FE$$

$$A_2 = \int_{1}^{2,5} (-x^3 + 2x^2 + 2,75x - 3,75)\,dx = \left[-\frac{1}{4}x^4 + \frac{2}{3}x^3 + 1,375x^2 - 3,75x\right]_{1}^{2,5}$$

$$= [-0,1] - [-0,6] = 0,5\ FE \qquad A_1 + A_2 = 5,8 + 0,5 = \underline{6,3\ FE}$$

(3) $x_1 = -2 \quad x_2 = 2$

$$A = \int_{-2}^{2} (-x^4 + 3x^2 + 4)\,dx = \left[-\tfrac{1}{5}x^5 + x^3 + 4x\right]_{-2}^{2}$$

$$= [9{,}6] - [-9{,}6] = \underline{19{,}2 \text{ FE}}$$

1c) $f(x) = \frac{1}{4}x^3 + x^2 \qquad x = 2 \qquad m = ?$

$f'(x) = \frac{3}{4}x^2 + 2x$

$f'(2) = \frac{3}{4} \cdot 2^2 + 2 \cdot 2 = 7 = m \qquad f(2) = 6$

$y = m \cdot x + b$

$6 = 7 \cdot 2 + b \qquad t(x) = 7x - 8$

$-8 = b$

2.) a) $f_1(x) = f_2(x)$

$x^3 + 1{,}5x^2 + 4 = 9x$

$x^3 + 1{,}5x^2 - 9x + 4 = 0 \qquad x_1 = 2$

Polynomdivision ergibt $\quad x_2 = 0{,}5 \quad x_3 = -4$

$f_2(2) = 18 \qquad S_1(2|18)$

$f_2(0{,}5) = 4{,}5 \qquad S_2(0{,}5|4{,}5)$

$f_2(-4) = -36 \qquad S_3(-4|-36)$

b) $f_1(x) = f_2(x)$

$0{,}2x^3 + 0{,}6x^2 - 2{,}6x - 3 = 2x^2 + 12x + 10$

$0{,}2x^3 - 1{,}4x^2 - 14{,}6x - 13 = 0$

$x^3 - 7x^2 - 73x - 65 = 0 \qquad x_1 = -1$

Polynomdiv. $\quad x_2 = 13$

$x_3 = -5$

$f_1(-1) = 0 \qquad S_1(-1|0)$

$f_2(13) = 504 \qquad S_2(13|504)$

$f_1(-5) = 0 \qquad S_3(-5|0)$

3.)

a) $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$(5|0) \rightarrow f(x)$ $f(5) = 0$ $\Rightarrow$ $0 = 125a + 25b + 5c + d$

berührt = Extr.

$x = 5$ Extr. $\rightarrow f'(x)$ $f'(5) = 0$ $\Rightarrow$ $0 = 75a + 10b + c$

$(3|-1) \rightarrow f(x)$ $f(3) = -1$ $\Rightarrow$ $-1 = 27a + 9b + 3c + d$

$x = 3$ WP $\rightarrow f''(x)$ $f''(3) = 0$ $\Rightarrow$ $0 = 18a + 2b$

b) AS $\Rightarrow$ nur gerade Exponenten

$f(x) = ax^4 + bx^2 + c$

$f'(x) = 4ax^3 + 2bx$

$(0|3) \rightarrow f(x)$ $f(0) = 3$ $\Rightarrow$ $c = 3$

$(-1|5) \rightarrow f(x)$ $f(-1) = 5$ $\Rightarrow$ $5 = a + b + c$

$x = -1$ HP $\rightarrow f'(x)$ $f'(-1) = 0$ $\Rightarrow$ $0 = -4a - 2b$

c) $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$x = -1$ Extr. $\rightarrow f'(x)$ $f'(-1) = 0$ $\Rightarrow$ $3a - 2b + c = 0$

$(-2|0) \rightarrow f(x)$ $f(-2) = 0$ $\Rightarrow$ $-8a + 4b - 2c + d = 0$

$(4|7) \rightarrow f(x)$ $f(4) = 7$ $\Rightarrow$ $64a + 16b + 4c + d = 7$

$x = 4$ $m = 10 \rightarrow f'(x)$ $f'(4) = 10$ $\Rightarrow$ $48a + 8b + c = 10$