Lösungen P12

① P12

Aufgabe 1

a) $P(x) = -2x + 18$

$P(x) = 0$

$0 = -2x + 18$

$2x = 18$

$x = 9$ ME

$D_{ök} = [0; 9]$

b) $E(x) = P(x) \cdot x$

$\underline{\underline{E(x) = -2x^2 + 18x}}$

$G(x) = E(x) - K(x)$

$G(x) = -2x^2 + 18x - (0{,}25x^3 - 2x^2 + 6x + 12{,}5)$

$G(x) = -2x^2 + 18x - 0{,}25x^3 + 2x^2 - 6x - 12{,}5$

$\underline{\underline{G(x) = -0{,}25x^3 + 12x - 12{,}5}}$

c) $G'(x) = 0$ und $G''(x) \neq 0$

$G'(x) = -0{,}75x^2 + 12$

$G''(x) = -1{,}5x$

$0 = -0{,}75x^2 + 12$

$0{,}75x^2 = 12$

$x^2 = 16 \quad |\sqrt{\ }$

$x_1 = 4$ ME x_{Gmax}

$G''(4) = -6 < 0 \Rightarrow \text{Max} \quad [x_2 = -4]$

$\underline{\underline{G(4) = 19{,}5 \text{ GE} \quad G_{max}}}$

$C(x_{Gmax} \mid P(x_{Gmax}))$

$x_{Gmax} = 4$ ME $\quad P(4) = 10$ GE

$\underline{\underline{C(4 \mid 10)}}$

d) $K'(x) = 0{,}75x^2 - 4x + 6$ Grenzkostenfunktion

$K''(x) = 1{,}5x - 4$

$K'''(x) = 1{,}5$

$K''(x) = 0$ und $K'''(x) \neq 0$

$0 = 1{,}5x - 4$

$4 = 1{,}5x \quad |:1{,}5$

$x = 2{,}7$ ME

$K'''(2{,}7) = 1{,}5 > 0 \Rightarrow$ Min

$K'(2{,}7) = 0{,}7$ GE

$\underline{\underline{GK_{min}(2{,}7|0{,}7)}}$

Aufgabe 2

a) $G(x) = -x^3 + 9x^2 - 2x - 48$

$G(x) = 0$

$0 = -x^3 + 9x^2 - 2x - 48 \quad |:(-1)$

$0 = x^3 - 9x^2 + 2x + 48$

$\underline{x_1 = 3 \text{ ME GS}}$

$(x^3 - 9x^2 + 2x + 48) : (x - 3) = x^2 - 6x - 16$

$-(x^3 - 3x^2)$

$-6x^2 + 2x$

$-(-6x^2 + 18x)$

$-16x + 48$

$-(-16x + 48)$

0

$x^2 - 6x - 16 = 0$

$x_{2/3} = +3 \pm \sqrt{9 + 16}$

$\underline{x_2 = 8 \text{ ME GG}}$

$[x_3 = -2]$

Gewinnzone $= GG - GS$

$= 8 - 3 = \underline{\underline{5 \text{ ME}}}$

b) $G'(x) = 0$ und $G''(x) \neq 0$

$G'(x) = -3x^2 + 18x - 2$

$G''(x) = -6x + 18$

$0 = -3x^2 + 18x - 2 \quad |:(-3)$

$0 = x^2 - 6x + \frac{2}{3}$

$x_{1/2} = +3 \pm \sqrt{9 - \frac{2}{3}}$

$x_1 = 5{,}9$ ME x_{Gmax} $\quad G''(5{,}9) = -17{,}4 < 0 \Rightarrow$ Max.

$x_2 = 0{,}1$ $\quad G''(0{,}1) = 17{,}4 > 0 \Rightarrow$ Min.

$\underline{\underline{G(5{,}9) = 48{,}1 \text{ GE} \quad G_{max}}}$

c) $E(x) = -3x^2 + 63x$

$p(x) = \frac{E(x)}{x}$

$p(x) = -3x + 63$

$p(x) = 0$

$0 = -3x + 63$

$3x = 63$

$x = 21$ ME SM

$\underline{\underline{D_{ök} = [0; 21]}}$

d) $G(9) = -66$ GE $\quad$ Es entsteht ein Verlust von 66 GE.

e) $G(x) = E(x) - K(x) \quad | + K(x)$

$G(x) + K(x) = E(x) \quad | - G(x)$

$K(x) = E(x) - G(x)$

$K(x) = -3x^2 + 63x - (-x^3 + 9x^2 - 2x - 48)$

$K(x) = -3x^2 + 63x + x^3 - 9x^2 + 2x + 48$

$K(x) = x^3 - 12x^2 + 65x + 48$

$\underline{\underline{K(10) = 498 \text{ GE}}}$

Aufgabe 3

a) $D_{ök} = [0; 7]$ $\quad$ 7 ME = SM $\Rightarrow (7|0)$

außerdem $(2|20)$

$E(x) = ax^2 + bx$

Gleichungssystem erstellen

I $\quad 0 = 49a + 7b \quad | \cdot 2$

II $\quad 20 = 4a + 2b \quad | \cdot (-7)$

$0 = 98a + 14b$

$-140 = -28a - 14b \quad \oplus$

$-140 = 70a \quad | : 70$

$\underline{\underline{-2 = a}}$

a in I

$0 = -98 + 7b$

$98 = 7b$

$\underline{\underline{14 = b}}$

$$\Rightarrow \quad \underline{\underline{E(x) = -2x^2 + 14x}}$$

$$\underline{\underline{P(x) = -2x + 14}}$$

b) $G(x) = E(x) - K(x)$

$G(x) = -2x^2 + 14x - (x^3 - 8x^2 - 3x + 50)$

$G(x) = -2x^2 + 14x - x^3 + 8x^2 + 3x - 50$

$\underline{G(x) = -x^3 + 6x^2 + 17x - 50}$

$G(x) = 0$

$0 = -x^3 + 6x^2 + 17x - 50 \quad | : (-1)$

$0 = x^3 - 6x^2 - 17x + 50$

$\underline{\underline{x_1 = 2\,ME\ GS}}$

$(x^3 - 6x^2 - 17x + 50) : (x - 2) = x^2 - 4x - 25$

$-(x^3 - 2x^2)$

$-4x^2 - 17x$

$-(-4x^2 + 8x)$

$-25x + 50$

$-(-25x + 50)$

0

$x^2 - 4x - 25 = 0$

$x_{2/3} = +2 \pm \sqrt{4 + 25}$

$\underline{\underline{x_2 = 7{,}4\,ME\ GG}}$

$[x_3 = -3{,}4]$

c) $G'(x) = 0$ und $G''(x) \neq 0$

$G'(x) = -3x^2 + 12x + 17$

$G''(x) = -6x + 12$

$0 = -3x^2 + 12x + 17 \quad | : (-3)$

$0 = x^2 - 4x - \frac{17}{3}$

$x_{1/2} = +2 \pm \sqrt{4 + \frac{17}{3}}$

$x_1 = 5{,}1\,ME \quad x_{Gmax}$

$[x_2 = -1{,}1]$

$G''(5{,}1) = -18{,}6 < 0 \Rightarrow Max$

$\underline{G(5{,}1) = 60{,}1\,GE}$

$\underline{G_{max}}$

5 P_{12}

d) $x_{Gmax} = 5{,}1\,ME$ $\quad p(x) = -2x+14$

$p(5{,}1) = 3{,}8\,GE$ $\quad \underline{\underline{C(5{,}1 \mid 3{,}8)}}$

Aufgabe 4

a) $p(x) = m\cdot x + b$ $\quad HP = 75\,GE$

$SM = 10\,ME \Rightarrow (10|0)$

$0 = m\cdot 10 + 75 \quad |-75$

$-75 = 10m \quad |:10$

$-7{,}5 = m$ $\quad \Rightarrow \underline{\underline{p(x) = -7{,}5x + 75}}$

$\underline{\underline{E(x) = -7{,}5x^2 + 75x}}$

b) $G(x) = E(x) - K(x)$

$G(x) = -7{,}5x^2 + 75x - (0{,}5x^3 - 7{,}5x^2 + 37{,}5x + 37)$

$G(x) = -7{,}5x^2 + 75x - 0{,}5x^3 + 7{,}5x^2 - 37{,}5x - 37$

$\underline{G(x) = -0{,}5x^3 + 37{,}5x - 37}$

$G(x) = 0 \quad 0 = -0{,}5x^3 + 37{,}5x - 37 \quad |:(-0{,}5)$

$0 = x^3 - 75x + 74$ $\quad \underline{\underline{x_1 = 1\,ME\ GS}}$

$(x^3 + 0x^2 - 75x + 74) : (x-1) = x^2 + 1x - 74$

$\underline{-(x^3 - 1x^2)}$

$1x^2 - 75x$

$\underline{-(1x^2 - 1x)}$

$-74x + 74$

$\underline{-(-74x + 74)}$

0

$x^2 + 1x - 74 = 0$

$x_{2/3} = -0{,}5 \pm \sqrt{0{,}25 + 74}$

$\underline{\underline{x_2 = 8{,}1\,ME\ GG}}$

$[x_3 = -9{,}1]$

c) $G'(x) = 0$ und $G''(x) \neq 0$

$G'(x) = -1{,}5x^2 + 37{,}5$ $\quad 0 = -1{,}5x^2 + 37{,}5$

$G''(x) = -3x$ $\quad 1{,}5x^2 = 37{,}5 \quad |:1{,}5$

$x^2 = 25 \quad |\sqrt{\ }$

$x_1 = 5\,ME$

$G''(5) = -15 < 0 \Rightarrow Max \quad [x_2 = -5]$

$\underline{\underline{x_{Gmax} = 5\,ME}}$

$P(5) = 37{,}5\,GE \quad \Rightarrow \quad \underline{\underline{C(5|37{,}5)}}$

d) $k(x) = 0{,}5x^3 - 7{,}5x^2 + 37{,}5x + 37$

$k'(x) = 1{,}5x^2 - 15x + 37{,}5$ $\qquad k''(x) = 0$ und $k'''(x) \neq 0$

$k''(x) = 3x - 15$

$k'''(x) = 3$

$0 = 3x - 15$

$-3x = -15 \;|:(-3)$

$x = 5\,ME$

$k'''(5) = 3 < 0$

$\Rightarrow Min$

$k'(5) = 0$

$\underline{\underline{Gk_{min}(5|0)}}$

Bei 5ME findet die geringste Kostensteigerung (von 0GE) statt, wenn man die ME verändert.