Lösungen P

1.

1.1. $K(x) = ax^3 + bx^2 + cx + d$ $\qquad K_v(x) = ax^3 + bx^2 + cx$

$K'(x) = 3ax^2 + 2bx + c$ $\qquad$ variable Kosten

$K_{fix} = 200,- €$ $\Rightarrow d = 200$

$(2|64) \rightarrow K_v(x)$ $\quad$ I $64 = 8a + 4b + 2c$ $\quad K_v(2) = 64$

$(1|244) \rightarrow K(x)$ $\quad$ II $244 = a + b + c + d$ $\quad K(1) = 244$

$(1|30) \rightarrow K'(x)$ $\quad$ III $30 = 3a + 2b + c$ $\quad K'(1) = 30$

d einsetzen in II

$244 = a + b + c + 200 \quad | -200$

II $44 = a + b + c$

$\Rightarrow$ I $64 = 8a + 4b + 2c$

II $44 = a + b + c \quad | \cdot (-2) \oplus$

III $30 = 3a + 2b + c \quad | \cdot (-1) \oplus$

IV $-24 = 6a + 2b$

V $14 = -2a - b \quad | \cdot 2$

IV $-24 = 6a + 2b$

V $28 = -4a - 2b \quad \oplus$

$4 = 2a \quad | : 2$

$\underline{\underline{2 = a}}$

$64 = 8a + 4b + 2c$

$-88 = -2a - 2b - 2c \quad \oplus$

IV $-24 = 6a + 2b$

$44 = a + b + c$

$-30 = -3a - 2b - c \quad \oplus$

V $14 = -2a - b$

a in V einsetzen

$14 = -2 \cdot 2 - b \quad | +4$

$18 = -b \quad | \cdot (-1)$

$\underline{\underline{-18 = b}}$

a und b einsetzen in II

$44 = 2 - 18 + c \quad | +16$

$\underline{\underline{60 = c}}$

$\underline{K(x) = 2x^3 - 18x^2 + 60x + 200}$

1.2. $K'(x) = 6x^2 - 36x + 60$

$K''(x) = 12x - 36$

$K'''(x) = 12$

$K''(x) = 0 \wedge K'''(x) \neq 0$

$0 = 12x - 36$

$36 = 12x$

$3 = x \qquad K'''(3) = 12 > 0 \Rightarrow Min$

$\underline{K'(3) = 6\,€} \qquad (3|6)\ GK_{min}$

1.3. $HP = 156\,€$

$SM = 13$ Stück

$p(x) = m \cdot x + b$

$0 = m \cdot 13 + 156 \qquad p(x) = -12x + 156$

$-156 = 13m \quad | : 13$

$-12 = m$

1.4. $G(x) = E(x) - K(x) \qquad E(x) = -12x^2 + 156x$

$G(x) = -12x^2 + 156x - (2x^3 - 18x^2 + 60x + 200)$

$= -12x^2 + 156x - 2x^3 + 18x^2 - 60x - 200$

$G(x) = -2x^3 + 6x^2 + 96x - 200$

$G(x) = 0$

$0 = -2x^3 + 6x^2 + 96x - 200 \quad | : (-2)$

$0 = x^3 - 3x^2 - 48x + 100 \qquad x_1 = 2\ \underline{GS}$

$(x^3 - 3x^2 - 48x + 100) : (x - 2) = x^2 - 1x - 50$

$-(x^3 - 2x^2)$

$-1x^2 - 48x$

$-(-1x^2 + 2x)$

$-50x + 100$

$-(-50x + 100)$

0

$x^2 - 1x - 50 = 0$

$x_{2/3} = +0{,}5 \pm \sqrt{0{,}25 + 50}$

$x_{2/3} = +0{,}5 \pm 7{,}1$

$x_2 = 7{,}6\ \underline{GG}$

$[x_3 = -6{,}6]$

1.5. $G'(x) = -6x^2 + 12x + 96$

$G''(x) = -12x + 12$

$G'(x) = 0 \wedge G''(x) \neq 0$

$0 = -6x^2 + 12x + 96 \quad | :(-6)$

$0 = x^2 - 2x - 16$

$x_{1/2} = +1 \pm \sqrt{1+16}$

$= +1 \pm 4{,}1$

$x_1 = 5{,}1$ $\qquad G''(5{,}1) = -49{,}2 < 0 \Rightarrow \text{Max}$

$[x_2 = -3{,}1]$ $\qquad \underline{G(5{,}1) = 180{,}4 \text{ €}}$

$p(5{,}1) = 94{,}8 \text{ €} \qquad \underline{C(5{,}1|94{,}8)}$

2.

2.1. $K(x) = ax^3 + bx^2 + cx + d$

$K'(x) = 3ax^2 + 2bx + c$

$K''(x) = 6ax + 2b$

$K(2) = 32$

$K'(1) = 6$

$K''(\frac{2}{3}) = 0$

$K_{fix} = 16 \text{ GE} \Rightarrow d = 16$

$(2|32) \rightarrow K(x)$ $\quad$ I $32 = 8a + 4b + 2c + d$

$(1|6) \rightarrow K'(x)$ $\quad$ II $6 = 3a + 2b + c$

$x = \frac{2}{3} \rightarrow K''(x)$ $\quad$ III $0 = 4a + 2b$

d einsetzen in I

$32 = 8a + 4b + 2c + 16 \quad | -16$

I $16 = 8a + 4b + 2c$

II $6 = 3a + 2b + c \quad | \cdot(-2)$] ⊕

III $0 = 4a + 2b$

$16 = 8a + 4b + 2c$

$-12 = -6a - 4b - 2c$] ⊕

$4 = 2a$

$\underline{2 = a}$

a einsetzen in III

$0 = 4 \cdot 2 + 2b \quad | -8$

$-8 = 2b$

$\underline{-4 = b}$

a und b in I

$16 = 8 \cdot 2 + 4 \cdot (-4) + 2c$

$16 = 2c \quad | :2$

$\underline{8 = c}$

$$\boxed{K(x) = 2x^3 - 4x^2 + 8x + 16}$$

2.2. $K'(x) = 6x^2 - 8x + 8$

$K''(x) = 12x - 8$

$K'''(x) = 12$

$K''(x) = 0 \wedge K'''(x) \neq 0$

$0 = 12x - 8$

$8 = 12x$

$\frac{2}{3} = x$ $\qquad K'''(\frac{2}{3}) = 12 > 0 \Rightarrow$ Min.

$K'(\frac{2}{3}) = \underline{5\frac{1}{3}\ GE}$ $\qquad (\frac{2}{3} | 5\frac{1}{3})$ GK$_{min}$

$(0{,}7 | 5{,}3)$

2.3. $(7|6)$ $\qquad p(x) = m \cdot x + b$

$(5|10)$

$6 = m \cdot 7 + b \quad | \cdot (-1)$

$10 = m \cdot 5 + b$

$-6 = -7m - b$

$10 = 5m + b$

$4 = -2m \quad | : (-2)$

$\underline{-2 = m}$

$p(x) = -2x + b$

$10 = -2 \cdot 5 + b \quad | + 10$

$\underline{20 = b}$

$p(x) = -2x + 20$

$p(x) = 0$

$0 = -2x + 20$

$2x = 20$

$\underline{x = 10\ ME}$

$p(x) \geq 0$

$\mathbb{D}_{ök} = [0; 10]$

3.

3.1. $f(x) = \frac{x^2 + 1}{x^2}$

1. $x^2 = 0$ $\qquad \mathbb{D} = \mathbb{R} \setminus \{0\}$

$x_{1/2} = 0$

2. $f(x) = 0$ $\qquad x^2 + 1 = 0 \Rightarrow x^2 = -1 \quad | \sqrt{\ }$

kein S_x

3. Keine behebbare Lücke
4. Poluntersuchung

$$\left.\begin{array}{l} l\text{-}\lim\limits_{x \to 0} \dfrac{x^2+1}{x^2} = +\infty \\[2ex] r\text{-}\lim\limits_{x \to 0} \dfrac{x^2+1}{x^2} = +\infty \end{array}\right\}$$ Pol ohne VZW

5. Asymptote $Zg = Ng \Rightarrow$ Polynomdivision

$$(x^2+1):(x^2) = 1 + \frac{1}{x^2}$$ Restglied

$$\frac{-(x^2)}{\quad +1}$$

$$\downarrow$$

$$y_A = 1$$

$$\lim_{x \to -\infty} \frac{1}{x^2} > 0$$ von oben

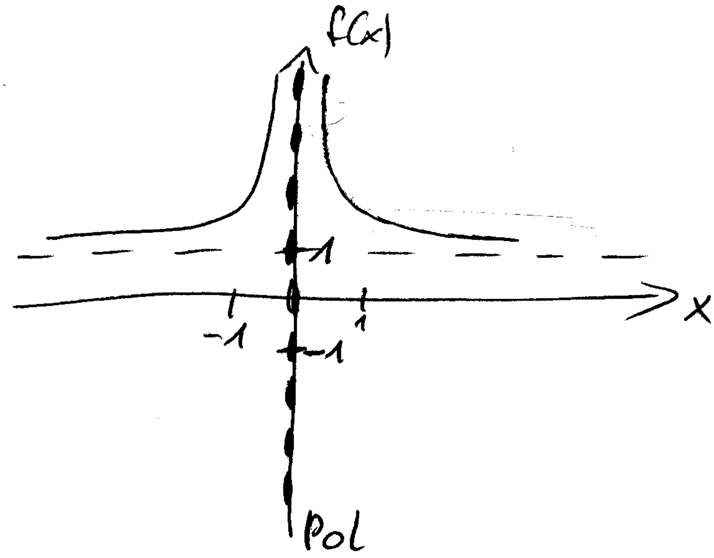
$$\lim_{x \to +\infty} \frac{1}{x^2} > 0$$ von oben

6. $f(0) = /$ da $\frac{0+1}{0}$ nicht definiert ist

Kein S_y

7. AS !

8. Skizze



3.2. $f(x) = \dfrac{x^2-4}{x^2-1}$

1. $x^2-1=0$

$x^2=1$

$x_1=+1$

$x_2=-1$

$\mathbb{D}=\mathbb{R}\setminus\{-1;1\}$

2. $f(x)=0$

$x^2-4=0$

$x^2=4$

$x_1=+2$

$x_2=-2$

nicht vorhanden

$S_{x_1}(+2|0)$

$S_{x_2}(-2|0)$

3. keine b. L.

4. Pole

$$\left.\begin{array}{l} \text{l-}\lim\limits_{x\to -1} \dfrac{x^2-4}{x^2-1} = -\infty \\ \text{r-}\lim\limits_{x\to -1} \dfrac{x^2-4}{x^2-1} = +\infty \end{array}\right\} \text{Pol mit VZW}$$

$$\left.\begin{array}{l} \text{l-}\lim\limits_{x\to +1} \dfrac{x^2-4}{x^2-1} = +\infty \\ \text{r-}\lim\limits_{x\to +1} \dfrac{x^2-4}{x^2-1} = -\infty \end{array}\right\} \text{Pol mit VZW}$$

5. Asymptote $Zg = Ng \Rightarrow$ Polynomdivision

$$(x^2-4):(x^2-1) = 1 - \frac{3}{x^2-1} \quad \text{Restglied}$$

$-(x^2-1)$

-3

$\downarrow$

$y_A = 1$

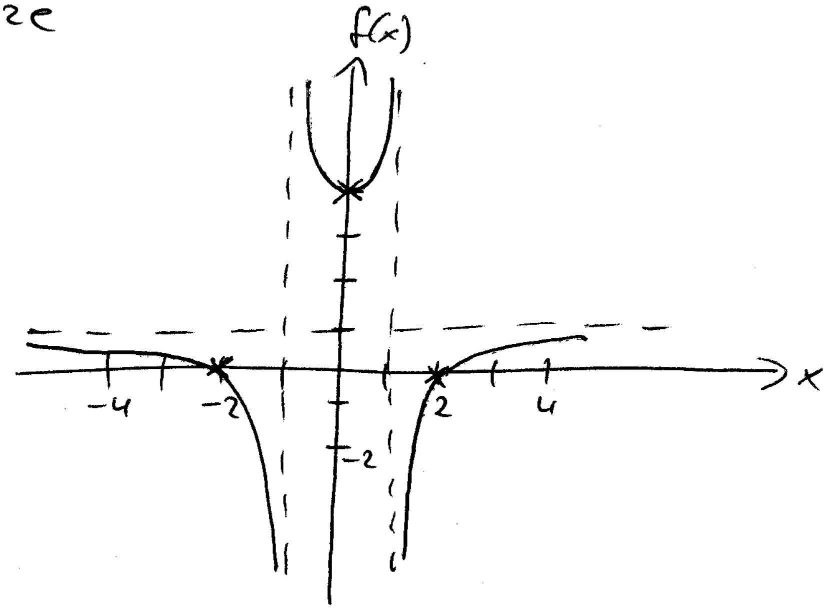
$\lim\limits_{x\to -\infty} \dfrac{-3}{x^2-1} < 0$ von unten

$\lim\limits_{x\to +\infty} \dfrac{-3}{x^2-1} < 0$ von unten

6. $f(0) = 4 \quad S_y(0|4)$

7. AS

8. Skizze



4.

4.1. $f(x) = ax^4 + bx^2 + c$ AS! $f'(x) = 4ax^3 + 2bx$

$(0|4) \rightarrow f(x) \quad f(0) = 4$ I $4 = c$

$(1|0) \rightarrow f(x) \quad f(1) = 0$ II $0 = a + b + c$

$x = 1; m = -6 \rightarrow f'(x) \quad f'(1) = -6$ III $-6 = 4a + 2b$

c einsetzen in II

$0 = a + b + 4 \quad | -4$

$-4 = a + b$

$-4 = a + b \quad | \cdot (-2)$

$-6 = 4a + 2b$

$\left.\begin{array}{l} +8 = -2a - 2b \\ -6 = 4a + 2b \end{array}\right] \oplus$

$2 = 2a$

$\underline{1 = a}$

a einsetzen in II

$-4 = 1 + b \quad | -1$

$\underline{-5 = b}$

$\underline{f(x) = x^4 - 5x^2 + 4}$

4.2.

$f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$f''(x) = 12ax^2 + 6bx + 2c$

$(0\mid-1) \rightarrow f(x)$	$f(0) = -1$	$-1 = e$	
$x=0$ Wp $\rightarrow f''(x)$	$f''(0) = 0$	$0 = 2c \Rightarrow c = 0$	
$x = 0; m = 2 \rightarrow f'(x)$	$f'(0) = 2$	$2 = d$	
$(2\mid0) \rightarrow f(x)$	$f(2) = 0$	I	$0 = 16a + 8b + 4c + 2d + e$
$x=2$ Extr. $\rightarrow f'(x)$	$f'(2) = 0$	II	$0 = 32a + 12b + 4c + d$

e, c, d einsetzen in I und II

$0 = 16a + 8b + 4 \cdot 0 + 2 \cdot 2 - 1$ $\qquad$ $0 = 32a + 12b + 4 \cdot 0 + 2$

$0 = 16a + 8b + 3 \quad | -3$ $\qquad$ $0 = 32a + 12b + 2 \quad | -2$

I $-3 = 16a + 8b$ $\qquad$ II $-2 = 32a + 12b$

$-3 = 16a + 8b \quad | \cdot (-2)$

$-2 = 32a + 12b$

$6 = -32a - 16b$

$-2 = 32a + 12b \quad \oplus$

$4 = -4b \quad | : (-4)$

$\underline{-1 = b}$

b einsetzen in I

$-3 = 16a + 8 \cdot (-1) \quad | +8$

$5 = 16a \quad | : 16$

$\frac{5}{16} = a$

$\underline{f(x) = \frac{5}{16}x^4 - x^3 + 2x - 1}$