Lösungen O

1.

1.1. $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$(-1|0) \to f(x)$ $\quad f(-1)=0 \quad 0 = -a+b-c+d$ I

$(-2|3) \to f(x)$ $\quad f(-2)=3 \quad 3 = -8a+4b-2c+d$ II

$x=-2; m=-2{,}5 \to f'(x)$ $\quad f'(-2)=-2{,}5 \quad -2{,}5 = 12a-4b+c$ III

$(0|-2) \to f(x)$ $\quad f(0)=-2 \quad -2 = d$ IV

$d = -2$ einsetzen in I und II

I $\quad 0 = -a+b-c-2$

II $\quad 3 = -8a+4b-2c-2$

III $\quad -2{,}5 = 12a-4b+c$

$\Rightarrow$

$2 = -a+b-c$

$5 = -8a+4b-2c$

$-2{,}5 = 12a-4b+c$ $\quad \oplus \quad |\cdot 2 \; \oplus$

$2 = -a+b-c$

$-2{,}5 = 12a-4b+c$

V $\quad -0{,}5 = 11a-3b$

$5 = -8a+4b-2c$

$-5 = 24a-8b+2c$

VI $\quad 0 = 16a-4b$

V $\quad -0{,}5 = 11a-3b \quad |\cdot(-4)$

VI $\quad 0 = 16a-4b \quad |\cdot 3$

$2 = -44a+12b$

$0 = 48a-12b$

$2 = 4a \quad |:4$

$\underline{0{,}5 = a}$

a einsetzen in VI

$0 = 16\cdot 0{,}5 - 4b$

$0 = 8-4b$

$4b = 8 \quad |:4$

$\underline{b = 2}$

a und b einsetzen in I

$2 = -0{,}5+2-c$

$2 = 1{,}5-c \quad |-1{,}5$

$0{,}5 = -c$

$\underline{-0{,}5 = c}$

$$\underline{f(x) = 0{,}5x^3 + 2x^2 - 0{,}5x - 2}$$

1.2. $f(x) = \frac{1}{2}x^3 + 2x^2 - \frac{1}{2}x - 2$

$f'(x) = \frac{3}{2}x^2 + 4x - \frac{1}{2}$

$f''(x) = 3x + 4$

$f'''(x) = 3$

1. Verlauf

$+$ vor $x^3 \Rightarrow$ geht nach oben

2. Symmetrie

Keine

3. $S_y(0|-2)$

$f(x) = 0$

$0 = \frac{1}{2}x^3 + 2x^2 - \frac{1}{2}x - 2 \quad | : \frac{1}{2}$

$0 = x^3 + 4x^2 - x - 4$

$x_1 = 1$

$$(x^3+4x^2-x-4):(x-1) = x^2+5x+4$$
$$\begin{array}{l} -(x^3-1x^2) \\ \hline \quad 5x^2 - x \\ \quad -(5x^2-5x) \\ \hline \qquad 4x-4 \\ \qquad -(4x-4) \\ \hline \qquad\quad 0 \end{array}$$

$x^2 + 5x + 4 = 0$

$x_{2/3} = -2{,}5 \pm \sqrt{6{,}25 - 4}$

$x_{2/3} = -2{,}5 \pm 1{,}5$

$x_2 = -1$

$x_3 = -4$

$S_{x_1}(1|0) \quad S_{x_2}(-1|0) \quad S_{x_3}(-4|0)$

4. Extremwerte

$f'(x) = 0 \wedge f''(x) \neq 0$

$0 = \frac{3}{2}x^2 + 4x - \frac{1}{2} \quad | : \frac{3}{2}$

$0 = x^2 + \frac{8}{3}x - \frac{1}{3}$

$$x_{1/2} = -\frac{4}{3} \pm \sqrt{\frac{16}{9} + \frac{1}{3}}$$

$$x_{1/2} = -\frac{4}{3} \pm 1{,}5$$

$x_1 = 0{,}2$ $\quad f''(0{,}2) = 4{,}6 > 0 \Rightarrow TP$

$x_2 = -2{,}8$ $\quad f''(-2{,}8) = -4{,}4 < 0 \Rightarrow HP$

$f(0{,}2) = -2$ $\quad TP(0{,}2 \mid -2)$ Das ist etwas schwierig zu zeichnen.

$f(-2{,}8) = 4{,}1$ $\quad HP(-2{,}8 \mid 4{,}1)$

5. Wendestellen

$f''(x) = 0 \wedge f'''(x) \neq 0$

$0 = 3x + 4 \quad | -4$

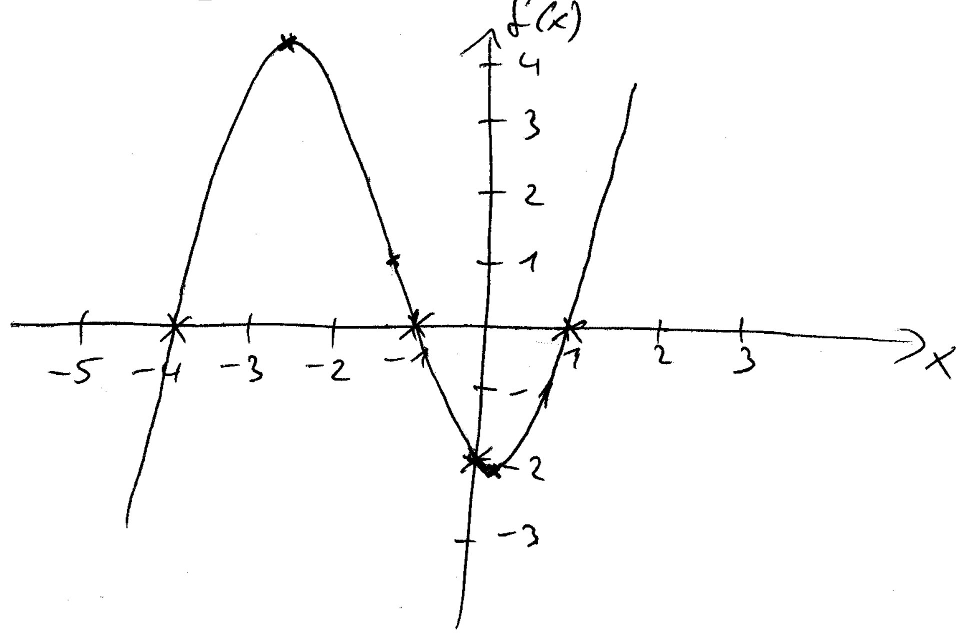
$-4 = 3x \quad | :3$

$-\frac{4}{3} = x$

$f'''(-\frac{4}{3}) = 3 > 0 \Rightarrow$ Rechts-Links-Krümmung

$f(-\frac{4}{3}) = 1$ $\quad W_{R-L}(-\frac{4}{3} \mid 1)$ oder $W_{RL}(-1{,}3 \mid 1)$

6. Skizze



1.3. $A_1 = \int_{-4}^{-1} \left(\frac{1}{2}x^3 + 2x^2 - \frac{1}{2}x - 2\right)dx$

$= \left[\frac{1}{8}x^4 + \frac{2}{3}x^3 - \frac{1}{4}x^2 - 2x\right]_{-4}^{-1}$

$= \left[1\frac{5}{24}\right] - \left[-6\frac{2}{3}\right] = 7\frac{7}{8}$ FE

$A_2 = \left|\int_{-1}^{1} \left(\frac{1}{2}x^3 + 2x^2 - \frac{1}{2}x - 2\right)dx\right|$

$= \left|\left[\frac{1}{8}x^4 + \frac{2}{3}x^3 - \frac{1}{4}x^2 - 2x\right]_{-1}^{1}\right|$

$= \left|\left[-1\frac{11}{24}\right] - \left[1\frac{5}{24}\right]\right| = \left|-2\frac{2}{3}\right| = 2\frac{2}{3}$ FE

$A_1 + A_2 = A_{ges.}$

$A_{ges} = 7\frac{7}{8} + 2\frac{2}{3} = 10\frac{13}{24} = \underline{\underline{10{,}5 \text{ FE}}}$

2.

2.1. $E(x) = -10x^2 + 300x$

$p(x) = -10x + 300$

$p(x) = 0$

$0 = -10x + 300$

$10x = 300$

$x = 30$

$p(x) \geq 0 \qquad D_{ök} = [0; 30]$

2.2. $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$(1|695) \rightarrow K(x)$ $\quad K(1) = 695$ $\quad$ I

(5)

Gk $(4|10) \rightarrow K'(x)$ $\quad K'(4) = 10$ $\quad$ II

$x = 4$ min. $\rightarrow K''(x)$ $\quad K''(4) = 0$ $\quad$ III

$K_{fix} = 500$

also $(0|500) \rightarrow K(x)$ $\quad K(0) = 500$ $\quad$ IV

↓

2wöchige Betriebsferien $\Rightarrow$ Kosten pro Woche 500GE!

I $695 = a + b + c + d$

II $10 = 48a + 8b + c$

III $0 = 24a + 2b$

IV $500 = d$

d einsetzen in I

$695 = a + b + c + 500 \quad | -500$

$195 = a + b + c \quad | \cdot (-1)$

$10 = 48a + 8b + c$

$-195 = -a - b - c$

$10 = 48a + 8b + c \quad \oplus$

$-185 = 47a + 7b$

$0 = 24a + 2b \quad | \cdot (-7)$

$-185 = 47a + 7b \quad | \cdot 2$

$0 = -168a - 14b$

$-370 = 94a + 14b$

$-370 = -74a \quad | : (-74)$

$+5 = a$

a einsetzen in III $\quad 0 = 24 \cdot 5 + 2b$

$-120 = 2b$

$-60 = b$

a und b einsetzen in I $\quad -195 = -5 + 60 - c \quad | -55$

$-250 = -c \quad | : (-1)$

$250 = c$

$K(x) = 5x^3 - 60x^2 + 250x + 500$

2.3. $G(x) = E(x) - K(x)$

$= -10x^2 + 300x - (5x^3 - 60x^2 + 250x + 500)$

$= -10x^2 + 300x - 5x^3 + 60x^2 - 250x - 500$

$G(x) = -5x^3 + 50x^2 + 50x - 500$

$G'(x) = -15x^2 + 100x + 50$

$G''(x) = -30x + 100$

$G'(x) = 0 \wedge G''(x) \neq 0$

$0 = -15x^2 + 100x + 50 \quad | :(-15)$

$0 = x^2 - \frac{20}{3}x - \frac{10}{3}$

$x_{1/2} = +\frac{10}{3} \pm \sqrt{\frac{100}{9} + \frac{10}{3}}$

$x_{1/2} = +\frac{10}{3} \pm 3{,}8$

$x_1 = 7{,}1$

$[x_2 = -0{,}5]$

$G''(7{,}1) = -113 < 0 \Rightarrow Max$

$p(7{,}1) = 229$ GE

$C(7{,}1 | 229)$

2.4. $K(5) = 875$ GE

$\frac{K(5)}{5} = \frac{875}{5} = 175$ GE pro (Stück) ME

2.5. $\frac{K(x)}{x}$ = Stückkosten

$\frac{K(x)}{x} = 50 \quad | \cdot x$

$K(x) = 50x \qquad E(x) = -10x^2 + 300x$

$G(x) = E(x) - K(x)$

$G(x) = -10x^2 + 300x - 50x$

$G(x) = -10x^2 + 250x$

3.

3.1. $f(x)=\frac{a(x+b)}{x+c}$

$x=-1$ Nst. $\rightarrow$ oben einsetzen 1.
$x=2$ Pol $\rightarrow$ unten einsetzen 2.
$S_y(0|-1)$ $\rightarrow$ a berechnen 3.

$f(x)=\frac{a(x+1)}{x-2}$ 1. 2.

$-1=\frac{a(0+1)}{0-2}$ 3.

$-1=\frac{a}{-2} \quad |\cdot(-2)$

$2=a$

$\Rightarrow f(x)=\frac{2(x+1)}{x-2}$

$f(x)=\frac{2x+2}{x-2}$

3.2. 1. $x-2=0$
$x=2$ $\quad \mathbb{D}=\mathbb{R}\setminus\{2\}$

2. $f(x)=0$
$2x+2=0$
$2x=-2$
$x=-1$ nicht vorhanden $\quad S_x(-1|0)$

3. Keine b.L.

4. Poluntersuchung
$\underset{x\to 2}{\text{l-Lim}}\ \frac{2x+2}{x-2}=-\infty$?

$\text{r-}\lim\limits_{x \to 2} \frac{2x+2}{x-2} = +\infty$ PoL mit VZW

5. Asymptote $Zg = Ng$ Polynomdivision

$(2x+2):(x-2) = 2 + \frac{6}{x-2}$

$-(2x-4)$

6

$\downarrow$

$y_A = 2$

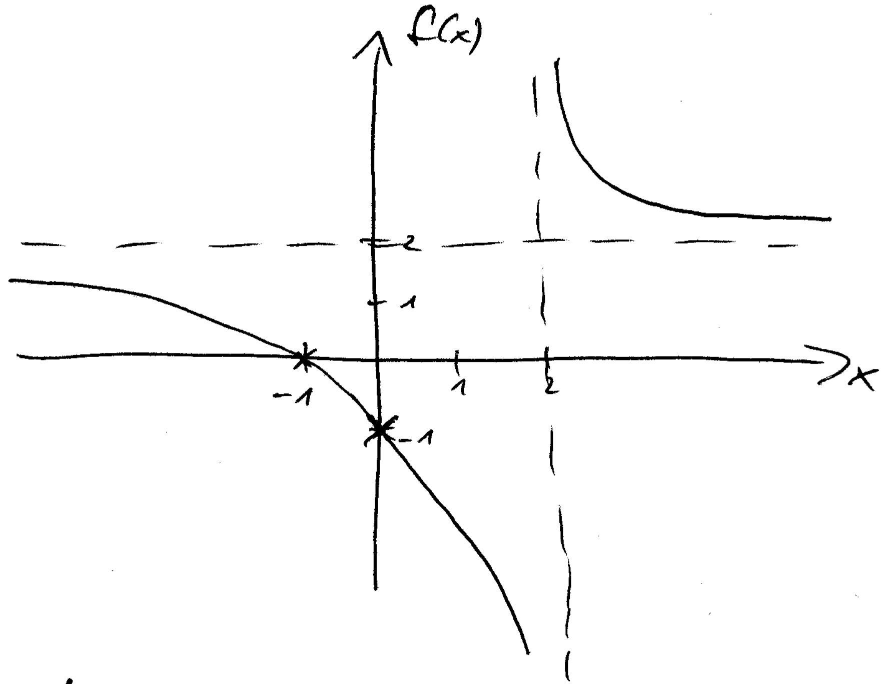
$\lim\limits_{x \to -\infty} \frac{6}{x-2} < 0 \Rightarrow$ von unten

$\lim\limits_{x \to +\infty} \frac{6}{x-2} > 0 \Rightarrow$ von oben

6. S_y $f(0) = -1$ $S_y(0|-1)$

7. KS

8. Skizze



3.3. $x=4$ Lücke $\Rightarrow$ oben und unten verändern

$$f(x) = \frac{(2x+2)(x-4)}{(x-2)(x-4)}$$

$$f(x) = \frac{2x^2-6x-8}{x^2-6x+8}$$