Wegen Platzsparung werden hier alle allgemeinen Gleichungen mit allen Ableitungen aufgeführt.

$f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$f''(x) = 12ax^2 + 6bx + 2c$

AS

$f(x) = ax^4 + bx^2 + c$

$f'(x) = 4ax^3 + 2bx$

$f''(x) = 12ax^2 + 2b$

$f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

PS

$f(x) = ax^3 + bx$

$f'(x) = 3ax^2 + b$

$f''(x) = 6ax$

Aufgabe 1

a)

ax^3

1. $f'(4) = 0$ I $0 = 48a + 8b + c$

2. $f''(1) = 0$ II $0 = 6a + 2b$

3. $f(2) = 3$ III $3 = 8a + 4b + 2c + d$

4. $f'(2) = -4$ IV $-4 = 12a + 4b + c$

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b)

ax^3

1. $f'(5) = -1$ I $-1 = 75a + 10b + c$

2. $f(1) = 4$ II $4 = a + b + c + d$

3. $f'(1) = 0$ III $0 = 3a + 2b + c$

4. $f''(4) = 0$ IV $0 = 24a + 2b$

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c)

ax^4

1. $f(3) = 5$ I $5 = 81a + 27b + 9c + 3d + e$

2. $f'(3) = 0$ II $0 = 108a + 27b + 6c + d$

3. $f'(-2) = -0,5$ III $-0,5 = -32a + 12b - 4c + d$

4. $f(0) = 0$ IV $0 = e$

5. $f''(0) = 0$ V $0 = 2c \Rightarrow c = 0$

d)

1. $f(-1) = -1$ I $-1 = a - b + c - d + e$

$\underline{ax^4}$ 2. $f''(-1) = 0$ II $0 = 12a - 6b + 2c$

3. $f(-3) = 0$ III $0 = 81a - 27b + 9c - 3d + e$

4. $f'(5) = \frac{1}{4}$ IV $\frac{1}{4} = 500a + 75b + 10c + d$

5. $f(5) = 0{,}65$ V $0{,}65 = 300a + 30b + 2c$

$t(5) = \frac{1}{4} \cdot 5 - 0{,}6$
$\qquad = 0{,}65$

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e)

1. $f(-2) = 1$ I $1 = 16a - 8b + 4c - 2d + e$

2. $f'(-2) = 0$ II $0 = -32a + 12b - 4c + d$

$\underline{ax^4}$ 3. $f''(4) = 0$ III $0 = 192a + 24b + 2c$

4. $f'(4) = -\frac{1}{2}$ IV $-\frac{1}{2} = 256a + 48b + 8c + d$

5. $f(4) = -4{,}5$ V $-4{,}5 = 256a + 64b + 16c + 4d + e$

$t(4) = -4{,}5$

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f)

1. $f(2) = -1$ I $-1 = 16a + 8b + 4c + 2d + e$

2. $f'(2) = 0$ II $0 = 32a + 12b + 4c + d$

$\underline{ax^4}$ 3. $f'(-2) = 0$ III $0 = -32a + 12b - 4c + d$

4. $f'(-3) = \frac{1}{6}$ IV $\frac{1}{6} = -108a + 27b - 6c + d$

5. $f(-3) = -3$ V $-3 = 81a - 27b + 9c - 3d + e$

$t(-3) = -3$

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g)

1. $f''(-2) = 0$ I $0 = 48a - 12b + 2c$

2. $f''(1) = 0$ II $0 = 12a + 6b + 2c$

$\underline{ax^4}$ 3. $f(2) = 4$ III $4 = 16a + 8b + 4c + 2d + e$

4. $f'(3) = -\frac{1}{3}$ IV $-\frac{1}{3} = 108a + 27b + 6c + d$

5. $f(3) = 3$ V $3 = 81a + 27b + 9c + 3d + e$

h)

ax^4

1. $f(-2) = -5$ I $-5 = 16a - 8b + 4c - 2d + e$

2. $f'(-2) = 0$ II $0 = -32a + 12b - 4c + d$

3. $f(1) = 0$ III $0 = a + b + c + d + e$

4. $f'(1) = 1{,}5$ IV $1{,}5 = 4a + 3b + 2c + d$

5. $f''(0) = 0$ V $0 = 2c$ $\Rightarrow$ $c = 0$

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i)

ax^6

1. $f(0) = 7$ I $7 = e$

2. $f(4) = 0$ II $0 = 256a + 64b + 16c + 4d + e$

3. $f''(2) = 0$ III $0 = 48a + 12b + 2c$

4. $f'(2) = -8$ IV $-8 = 32a + 12b + 4c + d$

5. $f'(-1) = 0$ V $0 = -4a + 3b - 2c + d$

Aufgabe 2

a)

ax^3

1. $f(6) = 0$ I $0 = 216a + 36b + 6c + d$

2. $f(1) = -1$ II $-1 = a + b + c + d$

3. $f(0) = 0$ III $0 = d$

4. $f'(0) = 0$ IV $0 = c$

c und d fällt weg $\Rightarrow$ I $0 = 216a + 36b$

     II $-1 = a + b$ $| \cdot (-36)$

a einsetzen in II

$-1 = 0{,}2 + b$ $| -0{,}2$

$-1{,}2 = b$

$\begin{aligned} 0 &= 216a + 36b \\ 36 &= -36a - 36b \end{aligned} \Big] \oplus$

$36 = 180a$ $| : 180$

$0{,}2 = a$

$\Rightarrow f(x) = 0{,}2x^3 - 1{,}2x^2$

b)

ax^3

1. $f(-1) = 0$ I $0 = -a + b - c + d$

2. $f(0) = 4$ II $4 = d$

3. $f'(0) = 0$ III $0 = c$

4. $f''(0) = 0$ IV $0 = 2b$ $\Rightarrow$ $b = 0$

b, c und d einsetzen in I

$$0 = -a + 0 - 0 + 4 \mid -4$$
$$-4 = -a$$
$$\underline{4 = a} \qquad \Rightarrow \underline{\underline{f(x) = 4x^3 + 4}}$$

c)

1. $f(0) = 0$ I $0 = d$

2. $f'(0) = 16$ II $16 = c$

$\underline{ax^3}$ 3. $f(4) = 0$ III $0 = 64a + 16b + 4c + d$

4. $f'(4) = 0$ IV $0 = 48a + 8b + c$

c und d einsetzen in III und IV

III $0 = 64a + 16b + 4 \cdot 16 + 0 \mid -64$

IV $0 = 48a + 8b + 16 \qquad\qquad \mid -16$

$$-64 = 64a + 16b$$
$$-16 = 48a + 8b \qquad \mid \cdot (-2)$$

$$-64 = 64a + 16b$$
$$32 = -96a - 16b \qquad \Big] \oplus$$

$$-32 = -32a \qquad \mid : (-32)$$
$$\underline{1 = a}$$

c und a einsetzen in IV

$$0 = 48 \cdot 1 + 8b + 16 \mid -64$$
$$-64 = 8b \qquad \mid : 8$$
$$\underline{-8 = b} \qquad\qquad \Rightarrow \underline{\underline{f(x) = x^3 - 8x^2 + 16x}}$$

d)

1. $f(0) = -2$ I $-2 = d$

2. $f'(0) = 0$ II $0 = c$

$\underline{ax^3}$ 3. $f'(-1) = -1{,}5$ III $-1{,}5 = 3a - 2b + c$

4. $f(1) = 0$ IV $0 = a + b + c + d$

c und d einsetzen in III und IV $-1{,}5 = 3a - 2b + 0$

$$0 = a + b + 0 - 2 \mid +2$$

$\underline{III}$ $\quad -1,5 = 3a - 2b$

$\underline{IV}$ $\quad 2 = a + b \quad |\cdot 2$

$\rule{3cm}{0.4pt}$

$\left.\begin{array}{l} -1,5 = 3a - 2b \\ 4 = 2a + 2b \end{array}\right] \oplus$

$\rule{3cm}{0.4pt}$

$2,5 = 5a \quad |:5$

$\underline{0,5 = a}$

a einsetzen in $\underline{IV}$

$2 = 0,5 + b \quad |-0,5$

$\underline{1,5 = b}$

$\Rightarrow \underline{\underline{f(x) = 0,5x^3 + 1,5x^2 - 2}}$

e)

1. $f(0) = 6 \qquad I \qquad 6 = d$

2. $f'(0) = -5 \qquad II \qquad -5 = c$

ax^3 3. $f(-1) = 8 \qquad III \qquad 8 = -a + b - c + d$

4. $f(3) = 0 \qquad IV \qquad 0 = 27a + 9b + 3c + d$

c und d einsetzen in III und IV

$8 = -a + b + 5 + 6 \qquad |-11$

$0 = 27a + 9b + 3 \cdot (-5) + 6 \quad |+9$

$\rule{5cm}{0.4pt}$

$-3 = -a + b \qquad |\cdot(-9)$

$9 = 27a + 9b$

$\rule{5cm}{0.4pt}$

$\left.\begin{array}{l} +27 = 9a - 9b \\ 9 = 27a + 9b \end{array}\right] \oplus$

$\rule{5cm}{0.4pt}$

$36 = 36a \quad |:36$

$\underline{1 = a}$

a einsetzen in III

$-3 = -1 + b \quad |+1$

$\underline{-2 = b}$

$\Rightarrow \underline{\underline{f(x) = x^3 - 2x^2 - 5x + 6}}$

f)

1. $f(0) = 0 \qquad I \qquad 0 = d$

2. $f'(0) = 0 \qquad II \qquad 0 = c$

ax^3 3. $f'(3) = -9 \qquad III \qquad -9 = 27a + 6b + c$

4. $f(6) = 0 \qquad IV \qquad 0 = 216a + 36b + 6c + d$

c und d fallen weg $\Rightarrow$

$-9 = 27a + 6b \quad |\cdot(-6)$

$0 = 216a + 36b$

$\rule{4cm}{0.4pt}$

$\left.\begin{array}{l} 54 = -162a - 36b \\ 0 = 216a + 36b \end{array}\right] \oplus$

$\rule{4cm}{0.4pt}$

$54 = 54a$

$\underline{1 = a}$

a einsetzen in III

$-9 = 27 \cdot 1 + 6b \quad |-27$

$-36 = 6b \quad |:6$

$$\underline{-6 = b} \quad \Rightarrow \underline{\underline{f(x) = x^3 - 6x^2}}$$

g)

ax^4

1.	$f(0) = -1$	I	$-1 = e$
2.	$f''(0) = 0$	II	$0 = 2c \Rightarrow c = 0$
3.	$f'(0) = 2$	III	$2 = d$
4.	$f(2) = 0$	IV	$0 = 16a + 8b + 4c + 2d + e$
5.	$f'(2) = 0$	V	$0 = 32a + 12b + 4c + d$

c, d und e einsetzen in IV und V

$0 = 16a + 8b + 4 \cdot 0 + 2 \cdot 2 - 1 \quad | -3$

$0 = 32a + 12b + 4 \cdot 0 + 2 \qquad | -2$

IV $\quad -3 = 16a + 8b \qquad | \cdot (-1,5)$

$\quad -2 = 32a + 12b$

$\quad 4,5 = -24a - 12b \quad \Big]\oplus$

$\quad -2 = 32a + 12b$

$\quad 2,5 = 8a$

$\quad \underline{\frac{5}{16} = a}$

a einsetzen in IV

$-3 = 16 \cdot \frac{5}{16} + 8b \, | -5$

$-8 = 8b \qquad | :8$

$\underline{-1 = b}$

$\Rightarrow \underline{\underline{f(x) = \frac{5}{16} x^4 - x^3 + 2x - 1}}$

h)

ax^4

1.	$f(0) = 0$	I	$0 = e$
2.	$f'(0) = 0$	II	$0 = d$
3.	$f''(0) = 0$	III	$0 = 2c \Rightarrow c = 0$
4.	$f(1) = -3$	IV	$-3 = a + b + c + d + e$
5.	$f'(1) = 0$	V	$0 = 4a + 3b + 2c + d$

c, d und e fallen weg $\Rightarrow$ IV $-3 = a + b \quad | \cdot (-3)$

V $\quad 0 = 4a + 3b$

$\quad 9 = -3a - 3b \quad \Big]\oplus$

$\quad 0 = 4a + 3b$

$\quad \underline{9 = a}$

a einsetzen in IV

$-3 = 9 + b \, | -9$

$\underline{-12 = b}$

$\Rightarrow \underline{\underline{f(x) = 9x^3 - 12x^2}}$