Lösungen M

1.

1.1. $f(x) = 0{,}25x^3 - 1{,}5x^2 + 8$

$f'(x) = 0{,}75x^2 - 3x$

$f''(x) = 1{,}5x - 3$

$f'''(x) = 1{,}5$

1. Verlauf

+ vor x^3 $\Rightarrow$ geht nach oben $\Rightarrow$ 

2. Symmetrie

ungerade und gerade Exponenten $\Rightarrow$ keine Symmetrie

3. S_x / S_y

$S_y(0|8)$

$f(x) = 0$

$0 = 0{,}25x^3 - 1{,}5x^2 + 8 \quad |:0{,}25$

$0 = x^3 - 6x^2 + 32$

$x_1 = 4$

$(x^3 - 6x^2 + 0x + 32) : (x - 4) = x^2 - 2x - 8$

$-(x^3 - 4x^2)$

$-2x^2 + 0x$

$-(-2x^2 + 8x)$

$-8x + 32$

$-(-8x + 32)$

0

$x^2 - 2x - 8 = 0$

$x_{2/3} = +1 \pm \sqrt{1+8}$

$x_{2/3} = +1 \pm 3$

$x_2 = 4$

$x_3 = -2$

$S_{x_{1/2}}(4|0) \quad S_{x_3}(-2|0)$

4. Extremwerte

$f'(x) = 0 \wedge f''(x) \neq 0$

$0 = 0{,}75x^2 - 3x \quad | : 0{,}75$

$0 = x^2 - 4x$

$0 = x(x-4)$

$x_1 = 0 \quad \wedge \quad x - 4 = 0$

$x_2 = 4$

$f''(0) = -3 < 0 \Rightarrow$ Hochpunkt

$f''(4) = +3 > 0 \Rightarrow$ Tiefpunkt

$f(0) = 8$ HP $(0|8)$

$f(4) = 0$ TP $(4|0)$

5. Wendepunkte

$f''(x) = 0 \wedge f'''(x) \neq 0$

$0 = 1{,}5x - 3 \quad | +3$

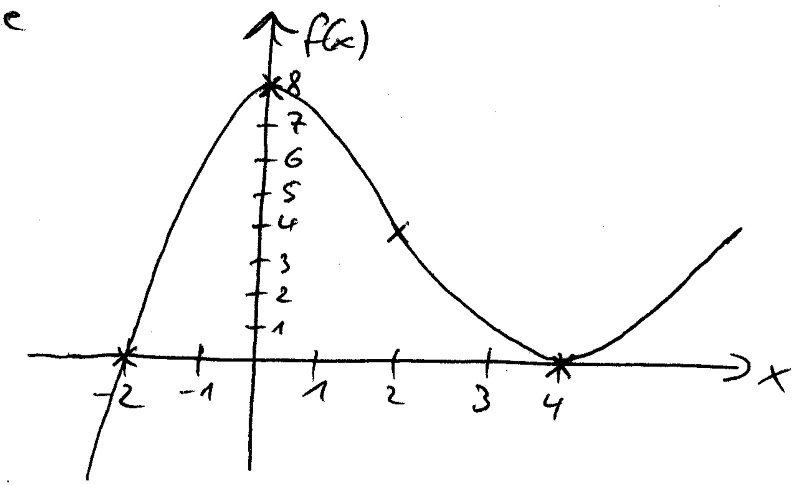
$3 = 1{,}5x \quad | : 1{,}5$

$2 = x$

$f'''(2) = 1{,}5 > 0 \Rightarrow$ Rechts-Links-Krümmung

$f(2) = 4$ $W_{R\text{-}L}(2|4)$

6. Skizze



1.2. $x = 1$

$f(1) = 6{,}75$ (y-Wert)

$f'(1) = -2{,}25$ (Steigung m)

$t(x) = m \cdot x + b$

$6{,}75 = -2{,}25 \cdot 1 + b \quad | +2{,}25$

$9 = b$

$t(x) = -2{,}25x + 9$

$t(x) = f(x)$ (Schnittpunkte)

$0{,}25x^3 - 1{,}5x^2 + 8 = -2{,}25x + 9$

$0{,}25x^3 - 1{,}5x^2 + 2{,}25x - 1 = 0 \quad | : 0{,}25$

$x^3 - 6x^2 + 9x - 4 = 0$

$x_1 = 1$ da hier Tangente anliegt

$(x^3 - 6x^2 + 9x - 4) : (x - 1) = x^2 - 5x + 4$

$-(x^3 - x^2)$

$-5x^2 + 9x$

$-(-5x^2 + 5x)$

$4x - 4$

$-(4x - 4)$

0

$x^2 - 5x + 4 = 0$

$x_{2/3} = +2{,}5 \pm \sqrt{6{,}25 - 4}$

$= +2{,}5 \pm 1{,}5$

$x_2 = 4$ neu

$x_3 = 1$ schon bekannt

$\underline{\underline{f(4) = 0}}$ $\Rightarrow$ In $P(4|0)$ trifft der Stein die Straße wieder.

1.3. $A = \int_{-2}^{4} (0{,}25x^3 - 1{,}5x^2 + 8)\,dx = \left[\frac{1}{16}x^4 - 0{,}5x^3 + 8x\right]_{-2}^{4}$

$= [16] - [-11] = \underline{\underline{27\ FE}}$

2.

2.1. $p(x) = m \cdot x + b$

$0 = m \cdot 32 + 9{,}6 \quad | -9{,}6$

$-9{,}6 = m \cdot 32 \quad | : 32$

$-0{,}3 = m$

$p(x) = -0{,}3x + 9{,}6$

$\underline{E(x) = -0{,}3x^2 + 9{,}6x}$

2.2. $G(x) = E(x) - K(x)$

$= -0{,}3x^2 + 9{,}6x - (0{,}2x^3 - 2{,}1x^2 + 7{,}8x + 16{,}2)$

$= -0{,}3x^2 + 9{,}6x - 0{,}2x^3 + 2{,}1x^2 - 7{,}8x - 16{,}2$

$\underline{G(x) = -0{,}2x^3 + 1{,}8x^2 + 1{,}8x - 16{,}2}$

$GS/GG \Rightarrow G(x) = 0$

$0 = -0{,}2x^3 + 1{,}8x^2 + 1{,}8x - 16{,}2 \quad | : (-0{,}2)$

$0 = x^3 - 9x^2 - 9x + 81$

$x_1 = 3 \quad \underline{\underline{GS}}$

$(x^3 - 9x^2 - 9x + 81) : (x-3) = x^2 - 6x - 27$

$-(x^3 - 3x^2)$

$-6x^2 - 9x$

$-(-6x^2 + 18x)$

$-27x + 81$

$-(-27x + 81)$

0

$x^2 - 6x - 27 = 0$

$x_{2/3} = +3 \pm \sqrt{9+27}$

$x_{2/3} = +3 \pm 6$

$x_2 = 9 \quad \underline{\underline{GG}}$

$[x_3 = -3]$

2.3. $G'(x) = 0 \wedge G''(x) \neq 0$

$G'(x) = -0{,}6x^2 + 3{,}6x + 1{,}8$

$G''(x) = -1{,}2x + 3{,}6$

(5)

$0 = -0{,}6x^2 + 3{,}6x + 1{,}8 \quad |:(-0{,}6)$

$0 = x^2 - 6x - 3$

$x_{1/2} = +3 \pm \sqrt{9+3}$

$x_{1/2} = +3 \pm 3{,}5$

$\underline{x_1 = 6{,}5}$ x_{Gmax}

$[x_2 = -0{,}5]$

$G''(6{,}5) = -4{,}2 < 0 \Rightarrow$ Max.

$p(6{,}5) = \underline{\underline{7{,}7\ GE}}$

$C(6{,}5 \mid 7{,}7)$

3.

3.1. $f(x) = \frac{x+2}{2x+2}$

1. $2x + 2 = 0 \quad |-2$

$2x = -2$

$x = -1 \quad \Rightarrow \mathbb{D} = \mathbb{R} \setminus \{-1\}$

2. $f(x) = 0$

$x + 2 = 0$

$x = -2$ nicht vorhanden

$S_x(-2 \mid 0)$

3. keine behebbare Lücke

4. Poluntersuchung

$\left.\begin{array}{l} \text{l-Lim}_{x \to -1 \ (-1{,}01)} \frac{x+2}{2x+2} \stackrel{+}{\underset{-}{=}} -\infty \\ \text{r-Lim}_{x \to -1 \ (-0{,}99)} \frac{x+2}{2x+2} \stackrel{+}{\underset{+}{=}} +\infty \end{array}\right\}$ Pol mit VZW

5. Asymptote $\quad Z_g = N_g \Rightarrow$ Polynomdivision

$(x+2) : (2x+2) = 0{,}5 + \frac{1}{2x+2}$ Restglied

$\frac{-(x+1)}{1}$

$y_A = 0{,}5$

Restgliedentersuchung

$$\lim_{x \to -\infty} \frac{1}{2x+2} < 0 \Rightarrow \text{von unten}$$

$$\lim_{x \to +\infty} \frac{1}{2x+2} > 0 \Rightarrow \text{von oben}$$

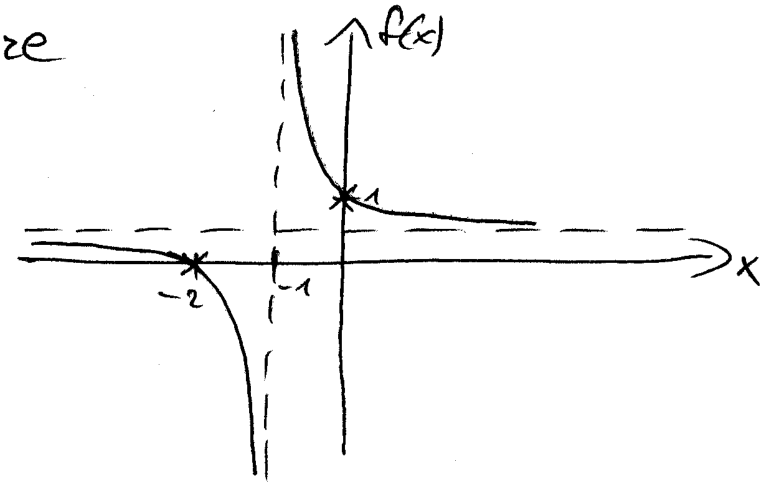
6. S_y

$f(0) = 1 \qquad S_y(0|1)$

7. Symmetrie

keine Symmetrie

8. Skizze



3.2. $x = 3$ Pol $\Rightarrow$ nur unten verändern

$$f(x) = \frac{(x+2)}{(2x+2)\cdot(x-3)}$$

$$f(x) = \frac{x+2}{2x^2-4x-6}$$

3.3. $x=3$ Lücke $\Rightarrow$ oben und unten verändern

$$f(x) = \frac{(x+2)(x-3)}{(2x+2)\cdot(x-3)}$$

$$f(x) = \frac{x^2-x-6}{2x^2-4x-6}$$