Lösungen L 12

Aufgabe 1

a) berührt bei -2 die x-Achse ⇒ $(-2|0)$ und Extremwert
$W(-1|-1)$ Punkt ; $x=-1$ Wendestelle

1. $f(-2)=0$	I $0=-8a+4b-2c+d$
2. $f'(-2)=0$	II $0=12a-4b+c$
3. $f(-1)=-1$	III $-1=-a+b-c+d$
4. $f''(-1)=0$	IV $0=-6a+2b$

$f(x)=ax^3+bx^2+cx+d$

$f'(x)=3ax^2+2bx+c$

$f''(x)=6ax+2b$

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b) $f(x)=ax^4+bx^3+cx^2+dx+e$
$f'(x)=4ax^3+3bx^2+2cx+d$

1. $(-2|0)$	$f(-2)=0$	I $0=16a-8b+4c-2d+e$
2. $x=-2$ Extr.	$f'(-2)=0$	II $0=-32a+12b-4c+d$
3. $(-1|3)$	$f(-1)=3$	III $3=a-b+c-d+e$
4. $x=-1$ Extr.	$f'(-1)=0$	IV $0=-4a+3b-2c+d$
5. $(1|0)$	$f(1)=0$	V $0=a+b+c+d+e$

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c) $f(x)=ax^3+bx^2+cx+d$
$f'(x)=3ax^2+2bx+c$

1. $x=3$ TP	$f'(3)=0$	I $0=27a+6b+c$
2. $(5|0)$	$f(5)=0$	II $0=125a+25b+5c+d$
3. $(-1|2)$	$f(-1)=2$	III $2=-a+b-c+d$
4. $x=-1$, $m=-0{,}5$	$f'(-1)=-0{,}5$	IV $-0{,}5=3a-2b+c$

d) $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

1. $x=3; m=2$	$f'(3)=2$	I $2 = 108a + 27b + 6c + d$
2. $(3\|5)$	$f(3)=5$	II $5 = 81a + 27b + 9c + 3d + e$
3. $(-1\|4)$	$f(-1)=4$	III $4 = a - b + c - d + e$
4. $x=-1; m=-2$	$f'(-1)=-2$	IV $-2 = -4a + 3b - 2c + d$
5. $(0\|0)$	$f(0)=0$	V $0 = e$

$t(3) = 2 \cdot 3 - 1$
$= 5$

6. $x=0$ TP $\quad f'(0)=0 \quad$ VI $0 = d$

Dies ist eine Angabe zu viel, aber hier gut zur Übung.

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e) Punktsymmetrie $\Rightarrow$ $f(x) = ax^5 + bx^3 + cx$

$f'(x) = 5ax^4 + 3bx^2 + c$

$f''(x) = 20ax^3 + 6bx$

1. $(2\|1)$	$f(2)=1$	I $1 = 32a + 8b + 2c$
2. $x=2; m=0$	$f'(2)=0$	II $0 = 80a + 12b + c$
3. $x=2$; WP	$f''(2)=0$	III $0 = 160a + 12b$

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f) Achsensymmetrie $\Rightarrow$ $f(x) = ax^4 + bx^2 + c$

$f'(x) = 4ax^3 + 2bx$

$f''(x) = 12ax^2 + 2b$

1. $(-3\|2)$	$f(-3)=2$	$2 = 81a + 9b + c$
2. $x=-3; m=-2$	$f'(-3)=-2$	$-2 = -108a - 6b$
3. $x=-3$; WP	$f''(-3)=0$	$0 = 108a + 2b$

Aufgabe 2

a) $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$f''(x) = 12ax^2 + 6bx + 2c$

1. $f(0) = 0$ I $0 = e$
2. $f'(0) = 0$ II $0 = d$
3. $f''(0) = 0$ III $0 = 2c \Rightarrow c = 0$
4. $f(2) = 0$ IV $0 = 16a + 8b + 4c + 2d + e$
5. $f'(1) = 2$ V $2 = 4a + 3b + 2c + d$

c, d, e fallen weg $\Rightarrow$ IV $0 = 16a + 8b$

V $2 = 4a + 3b \quad |\cdot(-4)$

$0 = 16a + 8b$

$-8 = -16a - 12b$ ⊕

$-8 = -4b$

$\underline{2 = b}$

b einsetzen in IV

$0 = 16a + 8 \cdot 2$

$0 = 16a + 16$

$-16 = 16a \quad |:16$

$\underline{-1 = a}$

$\Rightarrow \underline{\underline{f(x) = -1x^4 + 2x^3}}$ od. $\underline{\underline{f(x) = -x^4 + 2x^3}}$

b) PS $\Rightarrow f(x) = ax^3 + bx$

$f'(x) = 3ax^2 + b$

1. $f(1) = -8$ I $-8 = a + b$
2. $f'(0) = -9$ II $\underline{-9 = b}$

b einsetzen in I

$-8 = a - 9 \quad |+9$

$\underline{1 = a}$

$\Rightarrow \underline{\underline{f(x) = x^3 - 9x}}$

c) AS $\Rightarrow$ $f(x) = ax^4 + bx^2 + c$

$f'(x) = 4ax^3 + 2bx$

1. $f(0) = 4$	I	$4 = c$	c einsetzen in II
2. $f(1) = 0$	II	$0 = a + b + c$	$0 = a + b + 4 \quad \vert -4$
3. $f'(1) = -6$	III	$-6 = 4a + 2b$	II $-4 = a + b$

$$\begin{array}{lll} \text{II} & -4 = a + b & \vert \cdot (-2) \\ \text{III} & -6 = 4a + 2b & \\ \hline & 8 = -2a - 2b & \\ & -6 = 4a + 2b & \oplus \\ \hline & 2 = 2a & \\ & \underline{1 = a} & \end{array}$$

a einsetzen in II

$-4 = 1 + b$

$\underline{-5 = b}$

$$\Rightarrow \quad \underline{\underline{f(x) = x^4 - 5x^2 + 4}}$$

d) $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

1. $f'(2) = -6$	I	$-6 = 12a + 4b + c$
2. $f(0) = -1$	II	$-1 = d$
3. $f''(0) = 0$	III	$0 = 2b \Rightarrow b = 0$
4. $f'(0) = 6$	IV	$6 = c$

b und c einsetzen in I

$-6 = 12a + 4 \cdot 0 + 6 \quad \vert -6$

$-12 = 12a \quad \vert : 12$

$-1 = a$

$$\underline{\underline{f(x) = -x^4 + 6x - 1}}$$

e) $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

1. $f(0)=0$ I $0=d$
2. $f'(0)=0$ II $0=c$
3. $f(-3)=0$ III $0=-27a+9b-3c+d$
4. $f'(-3)=9$ IV $9=27a-6b+c$

c und d fallen weg $\Rightarrow$ III $0=-27a+9b$] ⊕
IV $9=27a-6b$

$9=3b$

$\underline{3=b}$

b einsetzen in III

$0=-27a+9\cdot3 \quad |-27$

$-27=-27a \quad |:(-27)$

$\underline{1=a}$

$\Rightarrow \underline{\underline{f(x)=x^3+3x^2}}$

f) $f(x)=ax^3+bx^2+cx+d$

$f'(x)=3ax^2+2bx+c$

$f''(x)=6ax+2b$

1. $f(4)=0$ I $0=64a+16b+4c+d$
2. $f'(4)=0$ II $0=48a+8b+c \quad |\cdot(-2)$
3. $f(2)=3$ III $3=8a+4b+2c+d \quad |\cdot(-1)$
4. $f''(2)=0$ IV $0=12a+2b \quad |\cdot(2)$

V $-3=56a+12b+2c$

VI $-3=-40a-4b$

I $0=64a+16b+4c+d$] ⊕
III $-3=-8a-4b-2c-d$

V $-3=56a+12b+2c$

II $0=-96a-16b-2c$] ⊕
V $-3=56a+12b+2c$

VI $-3=-40a-4b$

IV $0=24a+4b$] ⊕
VI $-3=-40a-4b$

$-3=-16a$

$\underline{\underline{\frac{3}{16}=a}}$

a einsetzen in IV $0=12\cdot\frac{3}{16}+2b$

$\underline{\underline{~~b=-\frac{9}{8}~~}}$

usw. einsetzen $\Rightarrow$ $c=0$ und $d=6$

$\Rightarrow \underline{\underline{f(x)=\frac{3}{16}x^3-\frac{9}{8}x^2+6}}$