Lösungen C 13

<u>1. Aufgabe</u>

a) $f(x)=\frac{2}{x-3}$

1.
$$N(x)=0$$
$$0=x-3 \quad |+3 \qquad D=R\setminus\{3\}$$
$$3=x$$

2.
$$f(x)=0$$
$$Z(x)=0$$
$$2\neq 0$$
kein S_x

$$f(0)=-\frac{2}{3} \qquad S_y(0|-0{,}7)$$

3. keine Ersatzfunktion

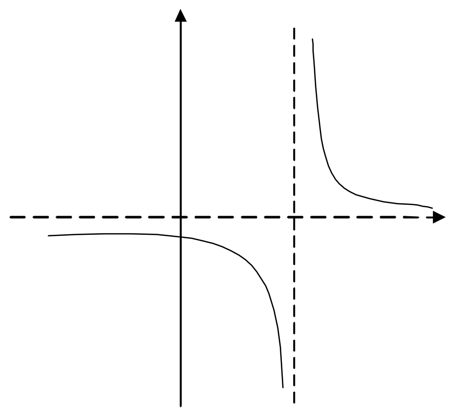
4. x = 3 ist Pol

$$\left.\begin{array}{ll} \text{l-lim}_{x\to 3} & \frac{2}{x-3}=-\infty \\ \text{r-lim}_{x\to 3} & \frac{2}{x-3}=+\infty \end{array}\right\} \text{Pol mit VZW}$$

5. Zg<Ng => $y_A = 0$ (x-Achse)

6. KS

7. Skizze



b) $f(x)=\frac{-3}{x+2}$

1.
$$N(x)=0$$
$$0=x+2 \quad |-2 \qquad D=R\setminus\{-2\}$$
$$-2=x$$

2.
$$f(x)=0$$
$$Z(x)=0$$
$$-3\neq 0$$
kein S_x

$$f(0)=-\frac{3}{2} \qquad S_y(0|-1{,}5)$$

3. keine Ersatzfunktion

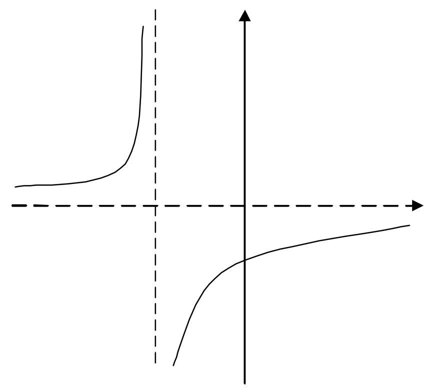
4. x = -2 ist Pol

$$\left.\begin{array}{ll} \text{l-lim}_{x\to -2} & \frac{-3}{x+2}=+\infty \\ \text{r-lim}_{x\to -2} & \frac{-3}{x+2}=-\infty \end{array}\right\} \text{Pol mit VZW}$$

5. Zg<Ng => $y_A = 0$ (x-Achse)

6. KS

7. Skizze



c) $f(x)=\frac{x^2-3x+2}{x^2-4}$

1.

$N(x)=0$

$0=x^2-4 \;|+4$ $\qquad D=R\setminus\{-2;2\}$

$4=x^2$

$x_1=2 \qquad x_2=-2$

2.

$f(x)=0$

$Z(x)=0$

$x^2-3x+2=0$ p-q-Formel

$x_1=2 \qquad x_2=1$

Kein S_x $\qquad S_x(1|0)$

$f(0)=-\frac{1}{2} \qquad S_y(0|-0{,}5)$

3. x = 2 ist Lücke

$f(x)=\frac{(x-2)(x-1)}{(x-2)(x+2)}$ (X-2) kürzen

$f^*(x)=\frac{x-1}{x+2}$ Ersatzfunktion zum Weiterarbeiten

4. x = -2 ist Pol

$$\left.\begin{array}{l}\text{l-lim}_{x\to -2}\ \frac{x-1}{x+2}=+\infty \\ \text{r-lim}_{x\to -2}\ \frac{x-1}{x+2}=-\infty\end{array}\right\}\text{Pol mit VZW}$$

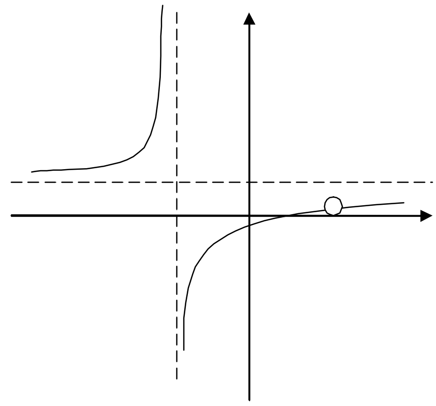
5. Zg=Ng => Polynomdivision

$$\begin{array}{l}(x-1):(x+2)=1-\frac{3}{x+2}\\ \underline{-(x+2)}\\ \quad -3\end{array}$$

$y_A = 1$

6. KS

7. Skizze



d) $f(x)=\frac{3x+6}{2x-6}$

1.

$N(x)=0$

$0=2x-6 \qquad D=R\setminus\{3\}$

$x=3$

2.

$f(x)=0$

$Z(x)=0$ $\qquad S_x(-2|0)$

$3x+6=0$

$x=-2$

$f(0)=-1 \qquad S_y(0|-1)$

3. keine Ersatzfunktion

4. x = 3 ist Pol

$$\left.\begin{array}{l}\text{l-lim}_{x\to 3}\ \frac{3x+6}{2x-6}=-\infty \\ \text{r-lim}_{x\to 3}\ \frac{3x+6}{2x-6}=+\infty\end{array}\right\}\text{Pol mit VZW}$$

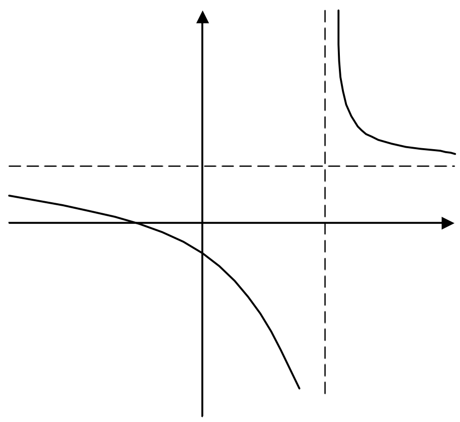
5. Zg=Ng => Polynomdivision

$$\begin{array}{l}(3x+6):(2x-6)=1{,}5+\frac{15}{2x-6}\\ \underline{-(3x-9)}\\ \quad 15\end{array}$$

$y_A = 1{,}5$

6. KS

7. Skizze



e) $f(x) = \dfrac{x^2 - 4x + 3}{x^2}$

1.

$N(x) = 0$

$0 = x^2$ $\qquad D = R \setminus \{0\}$

$x_{1/2} = 0$

2.

$f(x) = 0$

$Z(x) = 0$

$x^2 - 4x + 3 = 0$ p-q-Formel

$x_1 = 3$ $\qquad x_2 = 1$

$S_{x1}(3|0)$ $\qquad S_{x2}(1|0)$

$f(0) = /$ kein S_y

3. keine Ersatzfunktion

4. x = 0 ist Pol

$$\left.\begin{aligned} &\underset{x \to 0}{\text{l-lim}} \frac{x^2 - 4x + 3}{x^2} = +\infty \\ &\underset{x \to 0}{\text{r-lim}} \frac{x^2 - 4x + 3}{x^2} = +\infty \end{aligned}\right\} \text{Pol ohne VZW}$$

5. Zg=Ng => Polynomdivision

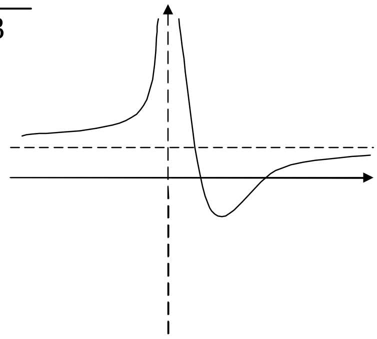
$$(x^2 - 4x + 3) : (x^2) = 1 - \frac{4x - 3}{x^2}$$

$$\frac{-(x^2)}{\quad -4x + 3}$$

$y_A = 1$

6. KS

7. Skizze



f) $f(x) = \dfrac{x}{x^3}$

1.

$N(x) = 0$ $\qquad D = R \setminus \{0\}$

$x = 0$

2.

$f(x) = 0$

$Z(x) = 0$ kein S_x

$x = 0$

$f(0) = /$ kein S_y

3. x = 0 ist Lücke

$f(x) = \dfrac{x}{x \cdot x \cdot x}$ x kürzen

$f^*(x) = \dfrac{1}{x^2}$

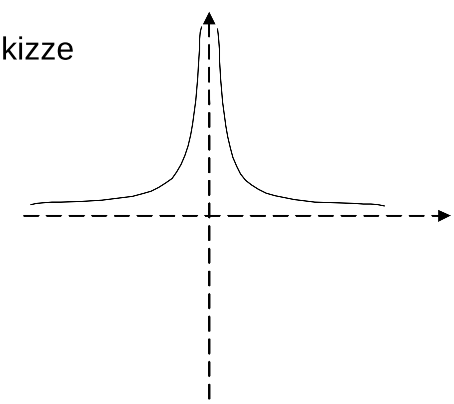
4. x = 0 ist Pol

$$\left.\begin{aligned} &\underset{x \to 0}{\text{l-lim}} \frac{1}{x^2} = +\infty \\ &\underset{x \to 0}{\text{r-lim}} \frac{1}{x^2} = +\infty \end{aligned}\right\} \text{Pol ohne VZW}$$

5. Zg<Ng => $y_A = 0$

6. KS

7. Skizze



g) $f(x) = \frac{2}{x^2 - 4}$

1.
$N(x) = 0$
$x^2 - 4 = 0$
$x_1 = 2$ V $x_2 = -2$

$D = R \setminus \{-2;2\}$

2.
$f(x) = 0$
$Z(x) = 0$
$2 \neq 0$ kein S_x

$f(0) = -\frac{1}{2}$ $S_y(0|-0{,}5)$

3. keine Ersatzfunktion

4. x = -2 ist Pol

$$\left.\begin{array}{l} \text{l-lim}_{x \to -2} \frac{2}{x^2 - 4} = +\infty \\ \text{r-lim}_{x \to -2} \frac{2}{x^2 - 4} = -\infty \end{array}\right\} \text{Pol mit VZW}$$

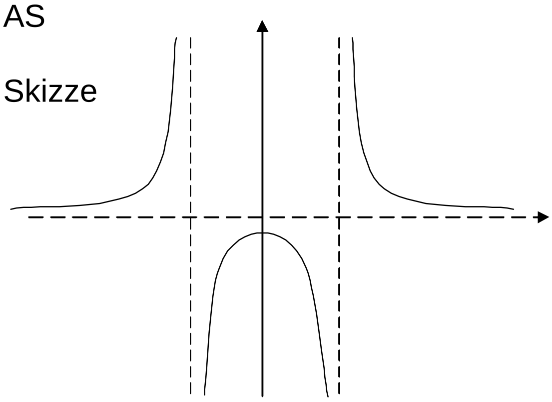
x = 2 ist Pol

$$\left.\begin{array}{l} \text{l-lim}_{x \to 2} \frac{2}{x^2 - 4} = -\infty \\ \text{r-lim}_{x \to 2} \frac{2}{x^2 - 4} = +\infty \end{array}\right\} \text{Pol mit VZW}$$

5. Zg<Ng => $y_A = 0$

6. AS

7. Skizze



h) $f(x) = \frac{x}{x^2 - 1}$

1.
$N(x) = 0$
$x^2 - 1 = 0$
$x_1 = 1$ V $x_2 = -1$

$D = R \setminus \{-1;1\}$

2.
$f(x) = 0$
$Z(x) = 0$ $S_x(0|0)$
$x = 0$
$f(0) = 0$ $S_y(0|0)$

3. keine Ersatzfunktion

4. x = -1 ist Pol

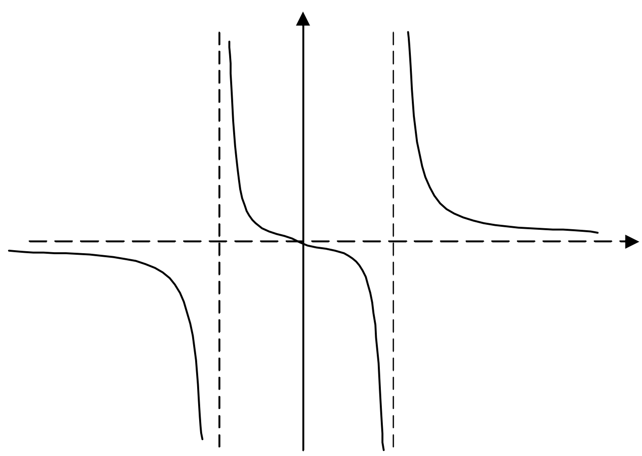
$$\left.\begin{array}{l} \text{l-lim}_{x \to -1} \frac{x}{x^2 - 1} = -\infty \\ \text{r-lim}_{x \to -1} \frac{x}{x^2 - 1} = +\infty \end{array}\right\} \text{Pol mit VZW}$$

x = 1 ist Pol

$$\left.\begin{array}{l} \text{l-lim}_{x \to 1} \frac{x}{x^2 - 1} = -\infty \\ \text{r-lim}_{x \to 1} \frac{x}{x^2 - 1} = +\infty \end{array}\right\} \text{Pol mit VZW}$$

5. Zg<Ng => $y_A = 0$

6. PS
7. Skizze



i) $f(x)=\frac{x^2+2}{x}$

1.
$N(x)=0$ $\quad D=R\ \backslash\{0\}$
$x=0$

2.
$f(x)=0$
$Z(x)=0$
$x^2+2=0$ kein S_x
$n.l.$
$f(0)=/$ kein S_y

3. keine Ersatzfunktion

4. x = 0 ist Pol

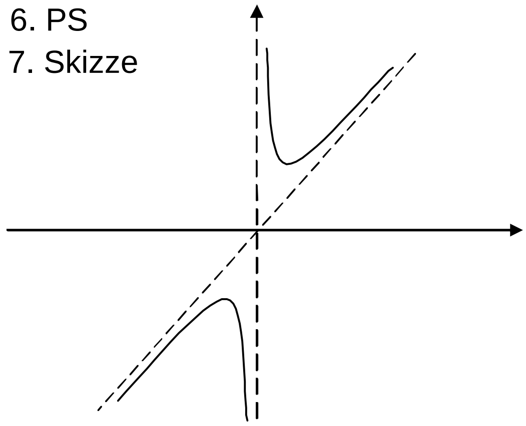
$$\left.\begin{array}{l} \text{l-lim}_{x\to 0}\ \frac{x^2+2}{x}=-\infty \\ \text{r-lim}_{x\to 0}\ \frac{x^2+2}{x}=+\infty \end{array}\right\}\ \text{Pol mit VZW}$$

5. Zg>Ng => Polynomdivision

$$\begin{array}{l} (x^2+2):x=x+\frac{2}{x} \\ \underline{-(x^2)} \\ \quad 0 \end{array}$$

$y_A = x$

6. PS
7. Skizze



<u>2. Aufgabe</u>

a) $f(x)=g(x)$ Schnittpunkte werden durch gleichsetzen berechnet

$\frac{3}{x-3}=3 \ |\cdot(x-3)$
$3=3\cdot(x-3)$
$3=3x-9 \ |+9$ Berechnung des zugehörigen y-Werts $f(4)=\frac{3}{4-3}$ $\quad S(3|4)$
$f(4)=3$
$12=3x \ |:3$
$4=x$

b) $f(x)=g(x)$

$\frac{4}{x-1}=x-1 \ |\cdot(x-1)$
$4=(x-1)\cdot(x-1)$
$4=x^2-2x+1 \ |-4$
$0=x^2-2x-3$
p-q-Formel ergibt
$x_1=3$ V $x_2=-1$

y-Werte $g(3)=2$ $\quad S_1(3|2)$
$g(-1)=-2$ $\quad S_2(-1|-2)$

c) $f(x)=g(x)$

$\frac{x-1}{x+3}=x-1 \ |\cdot(x+3)$
$x-1=(x-1)\cdot(x+3)$
$x-1=x^2+2x-3 \ |-x+1$
$0=x^2+x-2$
$x_1=1$ V $x_2=-2$

y-Werte $g(1)=0$ $\quad S_1(1|0)$
$g(-2)=-3$ $\quad S_2(-2|-3)$