Lösungen BW S18

1.Aufgabe

a) $f(x)=\frac{1}{2}x^3-x^2-\frac{5}{2}x$

D=R $x\to-\infty; f(x)\to-\infty$
$x\to+\infty; f(x)\to+\infty$

Keine Symmetrie, da gerade und ungerade Exponenten vorhanden sind.

$S_y(0|0)$

$f(x)=0$

$0=\frac{1}{2}x^3-x^2-\frac{5}{2}x$ normalisieren, x ausklammern, pq-Formel

$x_1=0;\ x_2\approx3{,}45;\ x_3\approx-1{,}45$

$S_{x1}(0|0)$ $S_{x2}(3{,}45|0)$ $S_{x3}(-1{,}45|0)$

$f'(x)=\frac{3}{2}x^2-2x-\frac{5}{2}$

$f''(x)=3x-2$

$f'''(x)=3$

$f'(x)=0$

$0=\frac{3}{2}x^2-2x-\frac{5}{2}$ normalisieren, pq-Formel

$x_1\approx2{,}12;\ x_2\approx-0{,}79$

$f'(x)=0\wedge f''(x)\neq0$

$f''(2{,}12)=4{,}36>0\Rightarrow T$

$f''(-0{,}79)=-4{,}37<0\Rightarrow H$

$f(2{,}12)\approx-5{,}03$ $f(-0{,}79)\approx1{,}10$

$T(2{,}12|-5{,}03)$ $H(-0{,}79|1{,}10)$

$f''(x)=0$

$0=3x-2$

$x=\frac{2}{3}$

$f''(x)=0\wedge f'''(x)\neq0$

$f'''\left(\frac{2}{3}\right)=3>0\Rightarrow R-L-K$

$f\left(\frac{2}{3}\right)\approx-1{,}96$

$W_{R-L}(0{,}67|-1{,}96)$

b) $x_1=-1$ und $t(x)=m\cdot x+b$

$f(-1)=1$ y-Wert

$f'(-1)=1$ Steigung m $f'(x)=m$

$1=1\cdot(-1)+b$

$b=2$

$t(x_1) = x + 2$

c) $f(x) = t(x_1)$

$\frac{1}{2}x^3 - x^2 - \frac{5}{2}x = x + 2$

$0{,}5x^3 - x^2 - 3{,}5x - 2 = 0$

TR: $x_{1/2} = -1$ Tangentenstelle und $x_3 = 4$

$t(4) = 6$

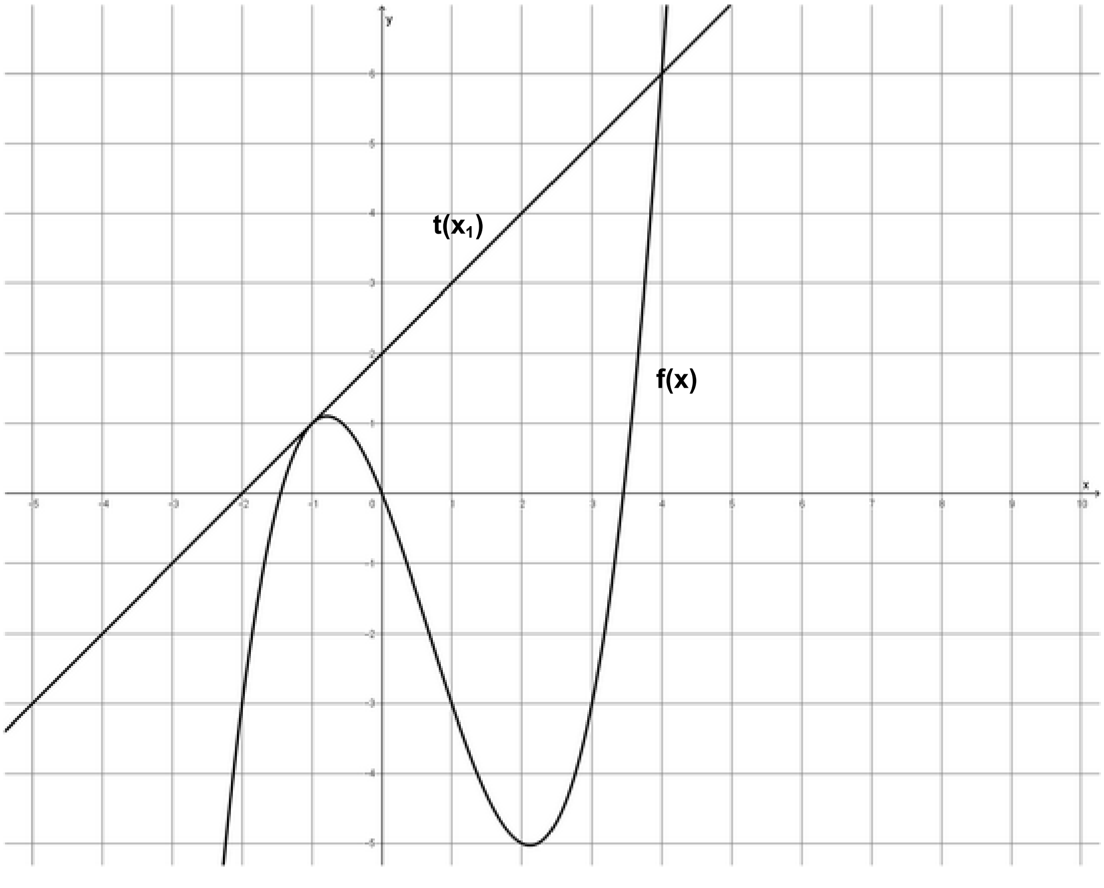
$S_3\left(4|6\right)$ weiterer Schnittpunkt

d) $f(-2) = -3$

$f(1) = -3$

$f(3) = -3$

e)



f) $A_1 = \int_{-1,45}^{0} \left(\frac{1}{2}x^3 - x^2 - \frac{5}{2}x\right) dx$

$A_1 = \left[\frac{1}{8}x^4 - \frac{1}{3}x^3 - \frac{5}{4}x^2\right]_{-1,45}^{0}$

$A_1 = [0] - [-1{,}06]$

$A_1 = 1{,}06 \text{FE}$

$A_2 = \left|\int_{0}^{3,45} \left(\frac{1}{2}x^3 - x^2 - \frac{5}{2}x\right) dx\right|$

$A_2 = \left|\left[\frac{1}{8}x^4 - \frac{1}{3}x^3 - \frac{5}{4}x^2\right]_{0}^{3,45}\right|$

$A_2 = \left|[-10{,}86] - [0]\right|$

$A_2 = |-10{,}86| = 10{,}86 \text{FE}$

$A = A_1 + A_2 = 1{,}06 + 10{,}86 = 11{,}92 \text{FE}$

g) $A=\left|\int_{-1}^{4}\left(\frac{1}{2}x^3-x^2-\frac{7}{2}x-2\right)dx\right|$ TR: $A=\left|-\frac{625}{24}\right|=\frac{625}{24}\text{FE}$

h) $[S_x t(x_1);S_1]$ => $t(x_1)=0$ => $x=-2$ => $[-2;-1]$

$A_1=\int_{-2}^{-1,45}(x+2)\,dx$

$A_1=\left[\frac{1}{2}x^2+2x\right]_{-2}^{-1,45}$

$A_1=\left[-\frac{1479}{800}\right]-[-2]$

$A_1=\frac{121}{800}\text{FE}$

$A_2=\left|\int_{-1,45}^{-1}\left(\frac{1}{2}x^3-x^2-\frac{7}{2}x-2\right)dx\right|$

$A_2=\left|\left[\frac{1}{8}x^4-\frac{1}{3}x^3-\frac{7}{4}x^2-2x\right]_{-1}^{-1,45}\right|$

$A_2=\left|[0,79]-\left[\frac{17}{24}\right]\right|$

$A_2=\left|-\frac{49}{600}\right|=\frac{49}{600}\text{FE}$

$$A=A_1+A_2=\frac{121}{800}+\frac{49}{600}=\frac{26}{75}\text{FE}$$

i) $A_2=\int_{-1}^{0}\left(\frac{1}{2}x^3-x^2-\frac{7}{2}x-2\right)dx$

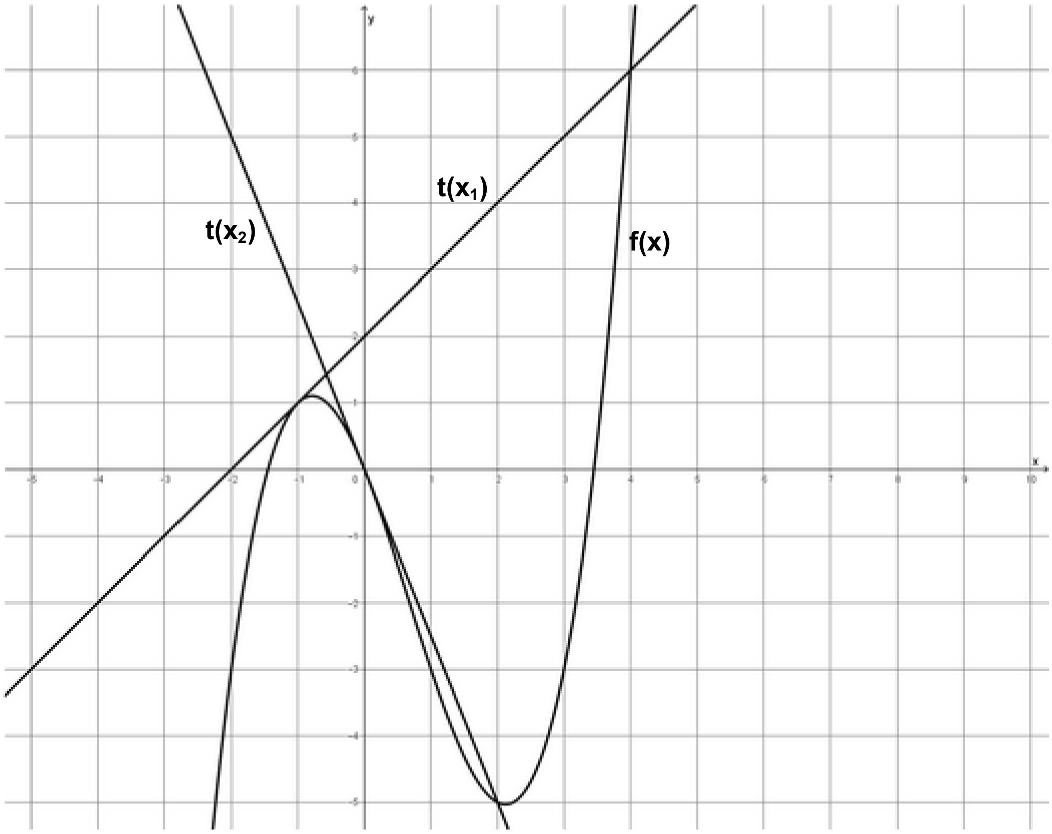
$A_3=\frac{17}{24}\text{FE}$

j) Tangente im Ursprung $(0|0)$ => $b=0$

$f'(x)=m$ mit $f'(0)=-2,5$

$t(x_2)=-2,5x$

k)



l) $f(x) = t(x_2)$

$$\frac{1}{2}x^3 - x^2 - \frac{5}{2}x = -\frac{5}{2}x$$

$$0{,}5x^3 - x^2 = 0$$

$$x^2(0{,}5x - 1) = 0$$

$x_{1/2} = 0$ Tangentenstelle und $x_3 = 2$

$$A = \int_0^2 \left(t(x_2) - f(x)\right) dx$$

$$A = \int_0^2 \left(-0{,}5x^3 + x^2\right) dx$$

$$A = \left[-\frac{1}{8}x^4 + \frac{1}{3}x^3\right]_0^2$$

$$A = \left[\frac{2}{3}\right] - [0]$$

$$A = \frac{2}{3}\text{FE}$$

m)