

Lösungen A 16

a) $x^3 - 4x^2 + x + 6 = 0$ Polynomdivision mit $x_1 = -1$

$$(x^3 - 4x^2 + x + 6) : (x + 1) = x^2 - 5x + 6$$

$$\underline{-(x^3 + 1x^2)}$$

$$-5x^2 + x$$

$$\underline{-(-5x^2 - 5x)}$$

$$6x + 6$$

$$\underline{-(6x + 6)}$$

$$0$$

$$x^2 - 5x + 6 = 0$$

pq-Formel

$$x_{2/3} = +2,5 \pm \sqrt{2,5^2 - 6}$$

$$x_2 = 3$$

$$x_3 = 2$$

b) $x^4 - 29x^2 + 100 = 0$ biquadratische Gleichung, Substitution

$$x^2 = z$$

$$z^2 - 29z + 100 = 0$$
 pq-Formel

$$z_{1/2} = +14,5 \pm \sqrt{14,5^2 - 100}$$

$$z_1 = 25$$

$$z_2 = 4$$

Resubstitution mit

$$z = x^2$$

$$x^2 = 25 \quad | \sqrt{\quad} \quad x_1 = 5 \quad ; \quad x_2 = -5$$

$$x^2 = 4 \quad | \sqrt{\quad} \quad x_3 = 2 \quad ; \quad x_4 = -2$$

c) $(x - 4)(x + 3) = (x + 5)^2 - 4$ Ausmultiplizieren der Klammern, binomische Formel

$$x^2 + 3x - 4x - 12 = x^2 + 10x + 25 - 4$$

$$x^2 - x - 12 = x^2 + 10x + 21 \quad | -x^2$$

$$-x - 12 = 10x + 21 \quad | -10x + 12$$

$$-11x = 33 \quad | :(-11)$$

$$x = -3$$

d) $x^3 - 5x^2 + 3x = 0$ Ausklammern von x

$$x \cdot (x^2 - 5x + 3) = 0$$

$$x_1 = 0 \quad ; \quad x^2 - 5x + 3 = 0 \quad \text{pq-Formel}$$

$$x_{2/3} = +2,5 \pm \sqrt{2,5^2 - 3}$$

$$x_2 = 4,3$$

$$x_3 = 0,7$$

e) $x^3 + 6x^2 = 0$ Ausklammern von x^2

$$x^2 \cdot (x + 6) = 0$$

$$x^2 = 0 \quad | \sqrt{\quad} \quad ; \quad x + 6 = 0 \quad | -6$$

$$x_{1/2} = 0 \quad ; \quad x_3 = -6$$

f) $x^4 - 81 = 0 \quad | + 81$
 $x^4 = 81 \quad | \sqrt[4]{}$
 $x_1 = 3 \quad ; \quad x_2 = -3$

g) $0,4x^3 + 1,2x^2 - 5,2x - 6 = 0$ Polynomdivision mit $x_1 = -1$
 $(0,4x^3 + 1,2x^2 - 5,2x - 6) : (x + 1) = 0,4x^2 + 0,8x - 6$
 $\underline{-(0,4x^3 + 0,4x^2)}$
 $\quad 0,8x^2 - 5,2x$
 $\quad \underline{-(0,8x^2 + 0,8x)}$
 $\quad \quad -6x - 6$
 $\quad \quad \underline{-(-6x - 6)}$
 $\quad \quad \quad 0$

$0,4x^2 + 0,8x - 6 = 0 \quad | : 0,4$
 $x^2 + 2x - 15 = 0$ pq-Formel
 $x_{2/3} = -1 \pm \sqrt{1^2 + 15}$
 $x_2 = 3$
 $x_3 = -5$

h) $-5x^2 + 20x = 0 \quad | : (-5)$
 $x^2 - 4x = 0$ Ausklammern von x
 $x(x - 4) = 0$
 $x_1 = 0 \quad ; \quad x_2 = 4$

i) $-\frac{1}{4}x^3 - x^2 - \frac{1}{4}x + 1,5 = 0 \quad | : (-\frac{1}{4})$
 $x^3 + 4x^2 + x - 6 = 0$ Polynomdivision mit $x_1 = 1$
 $(x^3 + 4x^2 + x - 6) : (x - 1) = x^2 + 5x + 6$
 $\underline{-(x^3 - x^2)}$
 $\quad 5x^2 + x$
 $\quad \underline{-(5x^2 - 5x)}$
 $\quad \quad 6x - 6$
 $\quad \quad \underline{-(6x - 6)}$
 $\quad \quad \quad 0$

$x^2 + 5x + 6 = 0$ pq-Formel
 $x_{2/3} = -2,5 \pm \sqrt{2,5^2 - 6}$
 $x_2 = -2$
 $x_3 = -3$

j) $-0,5x^4 + 5x^2 - 4,5 = 0 \quad | : (-0,5)$
 $x^4 - 10x^2 + 9 = 0$ Substitution
 $x^2 = z$
 $z^2 - 10z + 9 = 0$ pq-Formel
 $z_{1/2} = 5 \pm \sqrt{5^2 - 9}$
 $z_1 = 9$
 $z_2 = 1$
 $z = x^2$ Resubstitution
 $x^2 = 9 \quad | \sqrt{}$ $x_1 = 3 \quad ; \quad x_2 = -3$
 $x^2 = 1 \quad | \sqrt{}$ $x_3 = 1 \quad ; \quad x_4 = -1$

$$k) \quad \frac{1}{5}x^3 - x - 20 = 0 \quad | : \frac{1}{5}$$

$$x^3 - 5x - 100 = 0$$

Polynomdivision mit $x_1 = 5$

$$(x^3 + 0x^2 - 5x - 100) : (x - 5) = x^2 + 5x + 20 \quad \text{Platzhalter } + 0x^2 \text{ dazwischen setzen}$$

$$\begin{array}{r} -(x^3 - 5x^2) \\ \hline 5x^2 - 5x \\ -(5x^2 - 25x) \\ \hline 20x - 100 \\ -(20x - 100) \\ \hline 0 \end{array}$$

$$x^2 + 5x + 20 = 0 \quad \text{pq-Formel}$$

$$x_{2/3} = -2,5 \pm \sqrt{2,5^2 - 20}$$

nicht lösbar

$$l) \quad -x^2 + 81 = 0$$

$$x^2 = 81 \quad | \sqrt{\quad}$$

$$x_1 = 9$$

$$x_2 = -9$$

$$m) \quad 0,5x^4 - 8x^2 = 0$$

$$x^4 - 16x^2 = 0$$

Ausklammern von x^2

$$x^2(x^2 - 16) = 0$$

$$x_{1/2} = 0 \quad ; \quad x^2 - 16 = 0$$

$$x_3 = 4$$

$$x_4 = -4$$

$$n) \quad -3(x - 1) = -4(-2 - x) + 2 \quad \text{Klammern auflösen}$$

$$-3x + 3 = 8 + 4x + 2$$

$$-3x + 3 = 4x + 10 \quad | + 3x - 10$$

$$-7 = 7x \quad | : 7$$

$$x = -1$$

$$o) \quad \frac{5}{6}x + 15 = 0 \quad | - 15$$

$$\frac{5}{6}x = -15 \quad | : \frac{5}{6}$$

$$x = -18$$

$$p) \quad 3x^3 - 8,5x^2 - 4,5 = 0 \quad \text{Nicht durch Faktor 3 dividieren, da Brüche entstehen!}$$

Polynomdivision mit $x_1 = 3$

$$(3x^3 - 8,5x^2 + 0x - 4,5) : (x - 3) = 3x^2 + 0,5x + 1,5$$

$$\begin{array}{r} -(3x^3 - 9x^2) \\ \hline 0,5x^2 + 0x \\ -(0,5x^2 - 1,5x) \\ \hline 1,5x - 4,5 \\ -(1,5x - 4,5) \\ \hline 0 \end{array}$$

$$3x^2 + 0,5x + 1,5 = 0 \quad | : 3$$

$$x^2 + \frac{1}{6}x + 0,5 = 0 \quad \text{pq-Formel}$$

$$x_{2/3} = -\frac{1}{12} \pm \sqrt{\left(\frac{1}{12}\right)^2 - 0,5}$$

nicht lösbar

q) $-0,5x^2 - 4,5x - 4 = 0 \quad | :(-0,5)$

$x^2 + 9x + 8 = 0$ pq-Formel

$x_{1/2} = -4,5 \pm \sqrt{4,5^2 - 8}$

$x_1 = -1 \quad ; \quad x_2 = -8$

r) $\frac{3}{4}x^3 - \frac{1}{4}x^2 + \frac{3}{2}x - 2 = 0$ Nicht durch Faktor dividieren, da neue Brüche entstehen!

Polynomdivision mit $x_1 = 1$

$(\frac{3}{4}x^3 - \frac{1}{4}x^2 + \frac{3}{2}x - 2) : (x - 1) = \frac{3}{4}x^2 + \frac{1}{2}x + 2$

$$\begin{array}{r} \frac{3}{4}x^3 - \frac{3}{4}x^2 \\ \hline \frac{1}{2}x^2 + \frac{3}{2}x \\ -(\frac{1}{2}x^2 - \frac{1}{2}x) \\ \hline 2x - 2 \\ -(2x - 2) \\ \hline 0 \end{array}$$

$\frac{3}{4}x^2 + \frac{1}{2}x + 2 = 0 \quad | : \frac{3}{4}$

$x^2 + \frac{2}{3}x + \frac{8}{3} = 0$

pq-Formel

$x_{2/3} = -\frac{1}{3} \pm \sqrt{(\frac{1}{3})^2 - \frac{8}{3}}$

nicht lösbar

s) $4x^4 - 6x^3 = 0$

$x^3(4x - 6) = 0$ Ausklammern von x^3

$x^3 = 0 \quad | \sqrt[3]{\quad} \quad ; \quad 4x - 6 = 0$

$x_{1/2/3} = 0 \quad x_4 = 1,5$

t) $\frac{1}{2}x^3 - \frac{1}{4}x^2 - 7x = 0 \quad | : \frac{1}{2}$

$x^3 - 0,5x^2 - 14x = 0$ Ausklammern von x

$x(x^2 - 0,5x - 14) = 0$

$x_1 = 0 \quad ; \quad x^2 - 0,5x - 14 = 0$ pq-Formel

$x_{2/3} = 0,25 \pm \sqrt{0,25^2 + 14}$

$x_2 = 4$

$x_3 = -3,5$

u) $-3x^4 + 21x^2 - 36 = 0 \quad | :(-3)$

$x^4 - 7x^2 + 12 = 0$ Substitution

$x^2 = z$

$z^2 - 7z + 12 = 0$ pq-Formel

$z_{1/2} = 3,5 \pm \sqrt{3,5^2 - 12}$

$z_1 = 4$

$z_2 = 3$

$z = x^2$

$x^2 = 4 \quad | \sqrt{\quad} \quad x_1 = 2 \quad ; \quad x_2 = -2$

$x^2 = 3 \quad | \sqrt{\quad} \quad x_3 = 1,7 \quad ; \quad x_4 = -1,7$

v) $0,1x^3 - 0,2x^2 - 0,4x + 0,8 = 0 \mid : 0,1$

$$x^3 - 2x^2 - 4x + 8 = 0$$

Polynomdivision mit $x_1 = 2$

$$(x^3 - 2x^2 - 4x + 8) : (x - 2) = x^2 - 4$$

$$\begin{array}{r} -(x^3 - 2x^2) \\ \hline 0 - 4x + 8 \\ -(-4x + 8) \\ \hline 0 \end{array}$$

$$x^2 - 4 = 0 \mid + 4$$

$$x^2 = 4 \mid \sqrt{}$$

$$x_2 = 2 \text{ (doppelte Lösung)}$$

$$x_3 = -2$$

w) $-\frac{1}{5}x^4 + 2x^3 - 5x^2 = 0 \mid \left(-\frac{1}{5}\right)$

$$x^4 - 10x^3 + 25x^2 = 0$$

Ausklammern von x^2

$$x^2(x^2 - 10x + 25) = 0$$

$$x_{1/2} = 0 \quad ; \quad x^2 - 10x + 25 = 0 \quad \text{pq-Formel}$$

$$x_{3/4} = 5 \pm \sqrt{25 - 25}$$

$$x_{3/4} = 5$$

x) $x^4 - 18x^2 + 81 = 0$

Substitution

$$x^2 = z$$

$$z^2 - 18z + 81 = 0 \quad \text{pq-Formel}$$

$$z_{1/2} = 9 \pm \sqrt{81 - 81}$$

$$z_1 = 9$$

$$z_2 = 9$$

$$z = x^2$$

$$x^2 = 9 \mid \sqrt{}$$

$$x^2 = 9 \mid \sqrt{}$$

$$x_{1/3} = 3 \quad ; \quad x_{2/4} = -3$$

y) $x^4 - x^3 - 19x^2 - 11x + 30 = 0$

Polynomdivision mit $x_1 = 1$

$$(x^4 - x^3 - 19x^2 - 11x + 30) : (x - 1) = x^3 - 19x - 30$$

$$-(x^4 - x^3)$$

$$\hline -19x^2 - 11x$$

$$-(-19x^2 + 19x)$$

$$\hline -30x + 30$$

$$-(-30x + 30)$$

$$\hline 0$$

Polynomdivision mit $x_2 = -2$

$$(x^3 + 0x^2 - 19x - 30) : (x + 2) = x^2 - 2x - 15$$

$$-(x^3 + 2x^2)$$

$$\hline -2x^2 - 19x$$

$$-(-2x^2 - 4x)$$

$$\hline -15x - 30$$

$$-(-15x - 30)$$

$$\hline 0$$

$$x^2 - 2x - 15 = 0 \quad \text{pq-Formel}$$

$$x_{3/4} = 1 \pm \sqrt{1 + 15}$$

$$x_3 = 5 \quad x_4 = -3$$

z) $x^4 - 2,8x^3 - 11,48x^2 + 2,4x = 0$ Ausklammern von x

$x(x^3 - 2,8x^2 - 11,48x + 2,4) = 0$

$x_1 = 0$ $x^3 - 2,8x^2 - 11,48x + 2,4 = 0$

Polynomdivision mit $x_2 = 5$

$(x^3 - 2,8x^2 - 11,48x + 2,4) : (x - 5) = x^2 + 2,2x - 0,48$

$$\begin{array}{r}
 -(x^3 - 5x^2) \\
 \hline
 2,2x^2 - 11,48x \\
 -(2,2x^2 - 11x) \\
 \hline
 -0,48x + 2,4 \\
 -(-0,48x + 2,4) \\
 \hline
 0
 \end{array}$$

$x^2 + 2,2x - 0,48 = 0$

pq-Formel

$x_{3/4} = -1,1 \pm \sqrt{1,1^2 + 0,48}$

$x_3 = 0,2$ $x_4 = -2,4$