Lösungen A 14

a) $x^3 - 4x^2 + x + 6 = 0$ Polynomdivision mit $x_1 = -1$

$(x^3 - 4x^2 + x + 6) : (x+1) = x^2 - 5x + 6$

$-(x^3 + 1x^2)$

$-5x^2 + x$

$-(-5x^2 - 5x)$

$6x + 6$

$-(6x + 6)$

0

$x^2 - 5x + 6 = 0$ p-q-Formel

$x_{2/3} = +2{,}5 \pm \sqrt{2{,}5^2 - 6}$

$x_2 = 3$

$x_3 = 2$

b) $x^4 - 29x^2 + 100 = 0$ biquadratische Gleichung, Substitution mit

$x^2 = z$

$z^2 - 29z + 100 = 0$ p-q-Formel

$z_{1/2} = +14{,}5 \pm \sqrt{14{,}5^2 - 100}$

$z_1 = 25$

$z_2 = 4$

Resubstitution mit

$z = x^2$

$x^2 = 25 \ |\sqrt{\ }$ $x_1 = 5$; $x_2 = -5$

$x^2 = 4 \ |\sqrt{\ }$ $x_3 = 2$; $x_4 = -2$

c) $x^3 - 5x^2 + 3x = 0$ Ausklammern von x

$x \cdot (x^2 - 5x + 3) = 0$

$x_1 = 0$; $x^2 - 5x + 3 = 0$ p-q-Formel

$x_{2/3} = +2{,}5 \pm \sqrt{2{,}5^2 - 3}$

$x_2 = 4{,}3$

$x_3 = 0{,}7$

d) $x^3 + 6x^2 = 0$ Ausklammern von x^2

$x^2 \cdot (x + 6) = 0$

$x^2 = 0 \ |\sqrt{\ }$; $x + 6 = 0 \ |-6$

$x_{1/2} = 0$; $x_3 = -6$

e) $x^4 - 81 = 0 \ |+81$

$x^4 = 81 \ |\sqrt{\ }$

$x_1 = 3$; $x_2 = -3$

f) $0{,}4x^3+1{,}2x^2-5{,}2x-6=0$ Polynomdivision mit $x_1=-1$

$(0{,}4x^3+1{,}2x^2-5{,}2x-6):(x+1)=0{,}4x^2+0{,}8x-6$

$-(0{,}4x^3+0{,}4x^2)$

$0{,}8x^2-5{,}2x$

$-(0{,}8x^2+0{,}8x)$

$-6x-6$

$-(-6x-6)$

0

$0{,}4x^2+0{,}8x-6=0 \;\big|:0{,}4$

$x^2+2x-15=0$ p-q-Formel

$x_{2/3}=-1\pm\sqrt{1^2+15}$

$x_2=3$

$x_3=-5$

g) $-\frac{1}{4}x^3-x^2-\frac{1}{4}x+1{,}5=0 \;\big|:(-\frac{1}{4})$

$x^3+4x^2+x-6=0$ $x_1=1$

$(x^3+4x^2+x-6):(x-1)=x^2+5x+6$

$-(x^3-x^2)$

$5x^2+x$

$-(5x^2-5x)$

$6x-6$

$-(6x-6)$

0

$x^2+5x+6=0$ p-q-Formel

$x_{2/3}=-2{,}5\pm\sqrt{2{,}5^2-6}$

$x_2=-2$

$x_3=-3$

h) $-0{,}5x^4+5x^2-4{,}5=0 \;\big|:(-0{,}5)$

$x^4-10x^2+9=0$ Substitution

$x^2=z$

$z^2-10z+9=0$ p-q-Formel

$z_{1/2}=5\pm\sqrt{5^2-9}$

$z_1=9$

$z_2=1$

$z=x^2$

$x^2=9 \;\big|\sqrt{\;}$ $x_1=3$; $x_2=-3$

$x^2=1 \;\big|\sqrt{\;}$ $x_3=1$; $x_4=-1$

i) $\frac{1}{5}x^3-x-20=0 \;\big|:\frac{1}{5}$

$x^3-5x-100=0$ $x_1=5$

$(x^3+0x^2-5x-100):(x-5)=x^2+5x+20$

$-(x^3-5x^2)$

$5x^2-5x$

$-(5x^2-25x)$

$20x-100$

$-(20x-100)$

0

$x^2+5x+20=0$ p-q-Formel

$x_{2/3}=-2{,}5\pm\sqrt{2{,}5^2-20}$

nicht lösbar

j) $-x^2+81=0$

$x^2=81 \mid \sqrt{\ }$

$x_1=9$

$x_2=-9$

k) $0{,}5x^4-8x^2=0$

$x^4-16x^2=0$

$x^2(x^2-16)=0$

$x_{1/2}=0 \quad ; \quad x^2-16=0$

$x_3=4$

$x_4=-4$

l) $\frac{5}{6}x+15=0 \mid -15$

$\frac{5}{6}x=-15 \mid :\frac{5}{6}$

$x=-18$

m) $3x^3-8{,}5x^2-4{,}5=0 \mid \cdot 2$

$6x^3-17x^2-9=0$

$(6x^3-17x^2+0x-9):(x-3)=6x^2+x+3$

$-(6x^3-18x^2)$

x^2+0x

$-(x^2-3x)$

$3x-9$

$-(3x-9)$

0

Teilersuche schwierig, daher mal 2

$x_1=3$

$6x^2+x+3=0 \mid :6$

p-q-Formel

$x^2+\frac{1}{6}x+0{,}5=0$

$x_{2/3}=-\frac{1}{12}\pm\sqrt{\left(\frac{1}{12}\right)^2-0{,}5}$

nicht lösbar

n) $\frac{3}{4}x^3-\frac{1}{4}x^2+\frac{3}{2}x-2=0$

$x_1=1$

$(\frac{3}{4}x^3-\frac{1}{4}x^2+\frac{3}{2}x-2):(x-1)=\frac{3}{4}x^2+\frac{1}{2}x+2$

$-(\frac{3}{4}x^3-\frac{3}{4}x^2)$

$\frac{1}{2}x^2+\frac{3}{2}x$

$-(\frac{1}{2}x^2-\frac{1}{2}x)$

$2x-2$

$-(2x-2)$

0

$\frac{3}{4}x^2+\frac{1}{2}x+2=0 \mid :\frac{3}{4}$

p-q-Formel

$x^2+\frac{2}{3}x+\frac{8}{3}=0$

$x_{2/3}=-\frac{1}{3}\pm\sqrt{(\frac{1}{3})^2-\frac{8}{3}}$

nicht lösbar

o) $4x^4 - 6x^3 = 0$

$x^3(4x - 6) = 0$

$x^3 = 0 \ \left|\sqrt[3]{\ }\right.$; $4x - 6 = 0$

$x_{1/2/3} = 0$ $x_4 = 1{,}5$

p) $\frac{1}{2}x^3 - \frac{1}{4}x^2 - 7x = 0 \ \left|: \frac{1}{2}\right.$

$x^3 - 0{,}5x^2 - 14x = 0$

$x(x^2 - 0{,}5x - 14) = 0$

$x_1 = 0$; $x^2 - 0{,}5x - 14 = 0$ p-q-Formel

$x_{2/3} = 0{,}25 \pm \sqrt{0{,}25^2 + 14}$

$x_2 = 4$

$x_3 = -3{,}5$

q) $-3x^4 + 21x^2 - 36 = 0 \ \left|: (-3)\right.$

$x^4 - 7x^2 + 12 = 0$

$x^2 = z$

$z^2 - 7z + 12 = 0$ p-q-Formel

$z_{1/2} = 3{,}5 \pm \sqrt{3{,}5^2 - 12}$

$z_1 = 4$

$z_2 = 3$

$z = x^2$

$x^2 = 4 \ \left|\sqrt{\ }\right.$ $x_1 = 2$; $x_2 = -2$

$x^2 = 3 \ \left|\sqrt{\ }\right.$ $x_3 = 1{,}7$ $x_4 = -1{,}7$

r) $0{,}1x^3 - 0{,}2x^2 - 0{,}4x + 0{,}8 = 0 \left|: 0{,}1\right.$

$x^3 - 2x^2 - 4x + 8 = 0$ $x_1 = 2$

$(x^3 - 2x^2 - 4x + 8) : (x - 2) = x^2 - 4$

$-(x^3 - 2x^2)$

$0 - 4x + 8$

$-(-4x + 8)$

0

$x^2 - 4 = 0 \ \left|+4\right.$

$x^2 = 4 \left|\sqrt{\ }\right.$

$x_2 = 2$ (*doppelte Lösung*)

$x_3 = -2$

s) $-\frac{1}{5}x^4 + 2x^3 - 5x^2 = 0 \left| : \left(-\frac{1}{5}\right) \right.$

$x^4 - 10x^3 + 25x^2 = 0$

$x^2(x^2 - 10x + 25) = 0$

$x_{1/2} = 0 \quad ; \quad x^2 - 10x + 25 = 0$ p-q-Formel

$x_{3/4} = 5 \pm \sqrt{25 - 25}$

$x_{3/4} = 5$

t) $x^4 - 18x^2 + 81 = 0$

$x^2 = z$

$z^2 - 18z + 81 = 0$ p-q-Formel

$z_{1/2} = 9 \pm \sqrt{81 - 81}$

$z_1 = 9$

$z_2 = 9$

$z = x^2$

$x^2 = 9 \ |\sqrt{\ }$

$x^2 = 9 \ |\sqrt{\ }$

$x_{1/3} = 3 \quad ; \quad x_{2/4} = -3$